Volume-4, Issue-08, August 2017 **ISSN: 2349-7637 (Online)**

RESEARCH HUB – International Multidisciplinary Research Journal (RHIMRJ)

Research Paper

Analysis and discussion of a Sequences in Power Series

[1]Sukhen Bhattacharyya and [2]Rupen Chatterjee

[1]Research Scholar Department of Mathematics, Sri Satya Sai University of Technology & Medical Sciences, Sehore, Bhopal-Indore Road, Madhya Pradesh, India
[2]Department of Mathematics, Nabagram Hiralal Paul College, Nabagram, Hooghly, West Bengal, Pin:721246, India
(Affiliated to Calcutta University)

Abstract:
The rudimentary speculations of genuine esteemed elements of a genuine variable and of mathematical groupings and arrangement are treated in any standard analytics text. By and large, nonetheless, the confirmations are given in supplements and overlooked from the primary body of the course. To give thorough verifications of the essential hypotheses on combination, coherence, and differentiability, one necessities an exact meaning of genuine numbers. One approach to accomplish this is to begin with the development of genuine numbers from the reasonable ones by methods for Dedekind Cuts. We will not follow this way. All things considered, we will give a bunch of sayings for the genuine numbers from which every one of their properties can be concluded. These sayings will be isolated into three classes: First, we present the mathematical ones. Then, we examine the request maxims, lastly, we talk about the profound and key fulfillment saying. In the wake of illustrating the aphoristic meaning of the genuine numbers, we will take a gander at arrangements in $\mathbb{R}$ and their cutoff points. Here, the main idea is that of a Cauchy arrangement. It will be utilized in Appendix A for a concise conversation of Cantor's development of genuine numbers from the Cauchy arrangements in the set $\mathbb{Q}$ of levelheaded numbers. The properties of groupings will be utilized in a short area on boundless arrangement of genuine numbers. We will get back to limitless arrangement in another section to examine arrangement of capacities, for example, power arrangement and Fourier arrangement.

Key Words: Power series, mathematical, combination

1.Introduction

Sequence and series form an important component of Mathematical Analysis and arises in many situations. Infinite sequences and series were introduced briefly in connection with Zeno's paradoxes and the decimal representation of numbers. Many of the functions that arise in mathematical physics and chemistry, such as Bessel functions, Fourier series,Neumann series solution for integral equations etc. are defined as sums of series, so it is important to be familiar with the basic concepts of convergence of infinite sequences and series. The first rigorous treatment of sequence and series was made by Georg Cantor (1845-1918) and A. Cauchy (1789-1857).To develop a motion picture is very complex process and editing all the film into movie require that all the frames of action must be in order i.e., for mathematician frames are in a sequence. In sequence terms are arranged according to some definite pattern/rule and terms are separated by comma's and in case of series, terms are added (or subtracted or both).In fact,real sequence is function from set of natural number to set of real numbers i.e., f: $\mathbb{N}\rightarrow\mathbb{R}$ defined by f(x)= a_n. Listing the first few terms will not clearly indicate the sequence; in fact, sequence is a pattern of terms that are arranged in a meticulous way. The convergence of a sequence means that terms of sequence are merging towards a unique fixed finite number and divergence of sequence mean that terms are not approaching towards a fixed finite number. If the terms of sequence are approaching towards more than one finite number, we say that sequence is oscillating finitely and if the terms of sequence are approaching towards $+\infty$ and $-\infty$ both then we say sequence oscillating infinitely.

2.Objective

Our objective in this Chapter just as the two that follow is to discover a vigorous guess conspires for capacities. Specifically, we will perceive how to modify most capacities as such a limitless polynomial. We previously ventured out this in math I, we supplanted a capacity by its linearization. That is a first-request estimation. Next, you can supplant a capacity by a quadratic polynomial, this would be a second-request estimate. On the off chance that you proceed without end you show up at what is known as a force arrangement. By and by we can't go on always on a PC count, anyway we can keep the same number of terms as we have to show up at the exactness that the issue requires. This Chapter is expected to develop us to the point of comprehension the most effective method to painstakingly characterize a force arrangement. Verifiably the possibility of a force arrangement guess returns a few hundreds of years and improvements in analytics and arrangement/sequences have been inseparably connected. Sequences structure significant models in the investigation of cutoff points. Investigation (cautious arithmetic worked from restricting contentions) developed verifiably in light of the fact that it requested to show up at a sensibly steady treatment of sequences and arrangement. The better piece of the nineteenth century was loaded up with adjusting minor missteps in the contentions of Newton and Leibniz. Without getting excessively specialized, what happened was that the early fathers of math utilized force arrangement contentions without giving enough consideration to what the correct spaces ought to be for the arrangement. Subtleties and areas matter more when you begin getting to the edge of what is known.

In the nineteenth century cosmology accumulated perceptions of the movement of the planets that were exact. Notwithstanding, the arithmetic of Newton's Universal Law of Attraction didn't permit a definite arrangement. The issue of sorting out how all the planets pull on one another by the power of gravity is very muddled. There is the Sun and all the planets, their movements are coupled. Approximations to the real powers need tobe utilized just to make the science useful. Be that as it may, at that point you need to ensure the numerical estimate isn't making mistake bigger than the blunder inborn in the estimations themselves. It required a huge exertion by a multitude of mathematicians also, researchers to show that all the movements of the planets were clarified flawlessly by Newton's Theory. Well everything aside from the perihelion of Mercury. Turns out they determined accurately, Newton's hypothesis wasn't right. However, that is a story for one more day. Primary concern, power arrangement are a vital instrument for numerical sciences.

3.Sequence and Series

Sequence. A real sequence is a function f: N → R defined by f(n) $=u_n$.Now $f(1) = u_1, f(2) = u_2, f(3) = u_3$... $f(n) =$ u_n,where u_n is the n^{th}term of the sequence.

The sequence $u_1,u_2,u_3,\ldots ,u_n$ is designed briefly by $\{u_n\}$or $<u_n>$. The sequence is called finite or infinite according as the no of terms in sequence are finite or infinite.

If sequence is a function from set of natural numbers to set of complex numbers, the sequence is called complex valued sequence. Briefly, we can say that sequence is arrangement of terms according to some definite rule.Let k_1, k_2, k_3, k_4, k_5,... be positive integers such that $k_1<k_2<k_3<k_4<k_5\ldots$, then $\{u_{n_k}\}$is called a subsequence of $\{u_n\}$, n=1,2,3,....

Examples

1.The set of numbers 2, 7, 12, 17,...,32 is a finite sequence; the nth term isgiven by $u_n = 2 + 5(n-1) = 5n - 3$, n = 1,2, ... 7.

2. The set of numbers 1, 1/3 , 1/5, 1/7,.....is an infinite sequence with nth term $u_n = 1/(2n-1)$, n=1,2,3,...

Unless otherwise specified, we shall consider infinite sequences only.

Series

Suppose a frog has to cover I meter distance and as the frog starts jumping, it covers half distance in first jump and in second jump it again covers half way of the remaining distance, and continuing so on. Now question arises whether the frog ever cover the 1 meter distance, Mathematically, it seems that it is unending sum and does not cover 1 metre distance but actually it covers, this is possible by the limit concept.

$S_\infty = \frac{1}{2} + \frac{1}{2^2} + \frac{1}{2^3} \ldots + \frac{1}{2^n} + \cdots$ The limit of the sequence of partial sums is 1, i.e., $\lim_{n\to\infty} S_n = 1$.

The consideration of such type of sums launches us on the road to the theory of infinite series. The theory of infinite series, primarily as an extension of the theory of sequences.

If $\{u_n\}$ is a sequence of real numbers, then the expression

$u_1+u_2+\ldots+u_n+\ldots$.is called an infinite series. The infinite series $u_1+ u_2+\ldots+u_n+\ldots$. is denoted by $\sum_{n=1}^{\infty} u_n$ or $\sum u_n$.

Determine if the following series converges or diverges. If it converges find its value $\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}$.

Solution. Consider $u_n = \frac{1}{n^2+4n+3} = \frac{a}{n+1} + \frac{b}{n+3}$ (by partial fraction, we have a=$\frac{1}{2}$ $and\ b = -\frac{1}{2}$.

Now we can write $\frac{1}{n^2+4n+3} = \frac{\frac{1}{2}}{n+1} + \frac{-\frac{1}{2}}{n+3}$

$u_n = \frac{\frac{1}{2}}{n+1} + \frac{-\frac{1}{2}}{n+3} = \frac{1}{2}(\frac{1}{n+1} - \frac{1}{n+3})$.

$S_n = u_1+u_2+\ldots+u_{n-1}+u_n$ .Now on putting n=1, 2, 3,…,m and writing vertically and cancelling terms we have left with

$$S_n = \frac{1}{2}\left[\frac{1}{2} + \frac{1}{3} - \frac{1}{n+2} - \frac{1}{n+3}\right]$$

The limit of the partial sums is

$$\lim_{n\to\infty} S_n = \lim_{n\to\infty}\frac{1}{2}\left[\frac{1}{2}+\frac{1}{3}-\frac{1}{n+2}-\frac{1}{n+3}\right]=\frac{5}{12}$$

Hence the series is convergent (because the partial sums form a convergent sequence) and its value is $\frac{5}{12}$.

This part separates fiercely from all that we have done so far. Presently more than any time in recent memory it is significant that you not miss any talk. This section is considerably more about rationale and applying hypothesis than algorithmic figuring. For the greater part of you this isn't acceptable news. Nonetheless, don't surrender. Simply take it each day in turn and you'll get it. It will be simpler on the off chance that you have a decent demeanor about it. (I talk from my own insight)

A formal power series can be inexactly considered as an item that resembles a polynomial, yet with limitlessly numerous terms. Then again, for those acquainted with power series (or Taylor series), one may think about a formal power series as a power series where we disregard inquiries of assembly by not expecting that the variable X signifies any mathematical worth (not so much as an obscure worth). For instance, think about the series

$$A = 1 - 3X + 5X^2 - 7X^3 + 9X^4 - 11X^5 + \cdots.$$

On the off chance that we considered this as a power series, its properties would incorporate, for instance, that its range of assembly is 1. In any case, as a formal power series, we may overlook this totally; all that is significant is the grouping of coefficients.

power series

A *power series* is a series of the form

$$\sum_{n=0}^{\infty} c_n x^n = c_0 + c_1 x + c_2 x^2 + c_3 x^3 + \cdots$$

where x is a variable and the $c[n]$ are constants called the *coefficients* of the series. We can define the sum of the series as a function

$$f(x) = c_0 + c_1 x + c_2 x^2 + \cdots + c_n x^n + \cdots$$

with domain the set of all x for which the series converges.

More generally, a series of the form

$$\sum_{n=0}^{\infty} c_n (x-a)^n = c_0 + c_1 (x-a) + c_2 (x-a)^2 + \cdots$$

is called a *power series in (x-a)* or a *power series at a*. So, the question becomes "when does the power series converge?" Any of the series tests are available for use, but most often the Ratio Test is used. It tells us that the series converges when the limit of the ratio of the n+1st term to the nth term is less than one in absolute value, and diverges when the limit is greater than one in absolute value. In general, this boils down to

$$\lim_{n\to\infty}\left|\frac{c_{n+1}}{c_n}\right||x-a|.$$

When this limit is between -1 and 1, the series converges.

There are only three possibilities for how this series can converge:

- The series only converges at $x=a$
- The series converges for all x
- There is some positive number R such that the series converges for $|x-a|<R$ and diverges for $|x-a|>R$.

In the third case, R is called the *radius of convergence*. Note that the special cases of $|x-a|=R$ need to be checked separately. If the series only converges at a, we say the radius of convergence is zero, and if it converges everywhere, we say the radius of convergence is infinite.

For example, look at the power series

$$\sum_{n=0}^{\infty}\frac{1}{n}x^n$$

Using the ratio test, we find that

$$\lim_{n\to\infty}\frac{\frac{1}{n+1}}{\frac{1}{n}}x = \lim_{n\to\infty}\frac{nx}{n+1} = x$$

so the series converges when x is between -1 and 1. If x=1, then we get

$$\sum_{n=0}^{\infty}\frac{1}{n}$$

which diverges, since it is the harmonic series. If x=-1, then we get

$$\sum_{n=0}^{\infty}(-1)^n\frac{1}{n}$$

which converges, by the Alternating Series Test. So, the power series above converges for x in [-1,1).

One fact that may occasionally be helpful for finding the radius of convergence: if the limit of the nth root of the absolute value of $c[n]$ is K, then the radius of convergence is $1/K$.

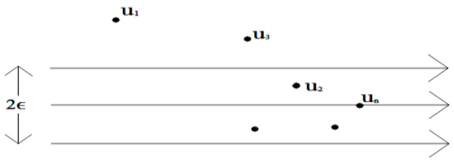
limit of a sequence (Convergence of sequence)

In literally way, the limit of a sequence is said to be*l* if all the terms of sequence after a fixed positive integer, say n, lies near around *l*. Mathematically, we define limit of a sequence as follow:

A number *l* is called the limit of an sequenceu_1,u_2,u_3,.... if for any positive number $\in>0$, we can find a positive number N depending on $\in$ such that

$|u_n - l|<\in$ for all integer n $\geq$**N**.

In such case, we write $\lim_{n\to\infty} u_n= l$. If the limit of a sequence exists (unique limit), then sequence is called convergent; otherwise, it may oscillate finitely (limits are finite but not unique) or infinitely (limits are not unique but are infinite) or it may diverge. Graphically,



A sequence if converges, then limit is unique.

In other words, we can say that the sequenceu_1,u_2,u_3,... has a limit *l* if the successive terms get “closer and closer” to *l*aftera fixed positive integers , say $n \geq N$i.e., terms of sequence lies very close to *l*i.e, the sequence $\{x_n\}$ converges to *l*, then *l*is called the limitof the sequence.

We note that there is a difference between a limit and a limit point. If x is the limit of $\{x_n\}$, then allterms of the sequence after a fixed positive integernliesarbitrarilyclose to x;

if pis a limit point of $\{x_n\}$, then infinitely many terms (all terms may lie or may not lie) approach arbitrarily close to p.A limit may be limit point but a limit point need not be limit of the sequence.

In fact, we do not define limit point of a sequence but, rather, a limit point of a range set.

Consider a sequence $\{a_n\}$, where a_n =1, the sequence converges to 1, but 1 is not a limit point of range set {1}, since there are no point in the nbd of 1 except 1.

4.General Concept

We start by talking about the idea of a sequence. Instinctively, a sequence is an arranged rundown of items or occasions. For example, the sequence of occasions at a crime location is significant for understanding the idea of the wrongdoing. In this course we will be keen on sequences of a more numerical nature; for the most part we will be keen on sequences of numbers, yet every so often we will think that its intriguing to think about sequences of focuses in a plane or in space, or on the other hand even sequences of sets

A geometric sequence has the form a, ar, ar2 , ar3 , ... for some fixed numbers a and r. An explicit formula for this geometric sequence is given by an = arn−1 , n ∈ N. A recursive formula is given by a1 = a and an = ran−1 for n > 1. Here are some examples of geometric sequences, see if you can determine a and r in each case:

2, 2, 2, 2, 2...
2, 4, 8, 16, 32, ...
3, 3/2, 3/4, 3/8, 3/16, ...
3, 1, 1/3, 1/9, 1/27, ..

Geometric sequences (with positive terms) are distinguished by the fact that the n th term is the geometric mean of its neighbors, i.e. an = √an+1an−1, (see exercise 13)

Example

If a batch of homebrew beer is inoculated with yeast it can be observed that the yeast population grows for the first several hours at a rate which is proportional to the population at any given time. Thus, if we let pn denote the yeast population measured after n hours have passed from the inoculation, we see that there is some number α > 1 so that

$$p_{n+1} = \alpha p_n.$$

That is, pn forms a geometric sequence. Actually, after a couple of days, the growth of the yeast population slows dramatically so that the population tends to a steady state. A better model for the dynamics of the population that reflects this behavior is

$$p_{n+1} = \alpha p_n - \beta p_n^2,$$

where α and β are constants determined by the characteristics of the yeast. This equation is known as the discrete logistic equation . Depending on the values of α and β it can display surprisingly varied behavior for the population sequence pn

Sequences of real numbers

Real number system

We know about common numbers and somewhat the reasonable numbers. While discovering roots of logarithmic conditions we see that sane numbers are adequately not to address roots which are not normal numbers. For instance draw the chart of y = x 2 − 2.

We see that it cross the x-pivot twice. The roots are with the end goal that their square is 2, yet they can't be judicious numbers as indicated by the accompanying theorem.

Theorem 1.1.1. Suppose that a0, a1, ..., an(n ≥ 1) are integers such that a0 6= 0, an 6= 0 and that r satisfies the equation

anx n + an−1x n−1 + ... + a1x + a0 = 0

If r = p q where p, q are integers with no common factors and q 6= 0. Then q divides an and p divides a0.

Proof: Since p q satisfies the equation, we have

anp n + an−1p n−1 q + ... + a0q n = 0

i.e., anp n = −q(an−1p n−1 + ... + a0q n−1). This means q divides an as p, q have no common factors. On the other hand we can also write

$$\frac{a_0 q^n}{p} = a_0 \frac{q_1^n \ldots q_k^n}{p_1 \ldots p_j}$$

Now we see that the possible rational roots of x 2 − 2 = 0 are ±1, ±2. But it is easy to check that ±1, ±2 does not satisfy x 2 − 2 = 0. So the roots of x 2 − 2 = 0 are not rational numbers. This means the set of rational numbers has "gaps". So the natural question to ask is: Can we have a number system without these gaps? The answer is yes and the "complete number system" without these gaps is the real line R. We will not look into the development of R as it is not easy to define the real numbers. We assume that there is a set R, whose elements are called real numbers and R is closed with respect to addition and multiplication. That is, given any a, b ∈ R, the sum a + b and product ab also represent real numbers. Moreover, R has an order structure ≤ and has no "gaps" in the sense that it satisfies the Completeness Axiom(see below). Let S be a non-empty subset of R. If S contains a largest element s 0 , then we call s 0 the maximum of S. If S contains a smallest element s0, then we call s0 the minimum of S. If S is bounded above and S has least upper bound, then we call it the supremum of S. If S is bounded below and S has greatest lower bound, then we call it as infimum of S. Unlike maximum and minimum, sup S and inf S need not belong to the set S. An important observation is if α = sup S is finite, then for every > 0, there exists an element s ∈ S such that s ≥ α − . Note that any bounded subset of Natural numbers has maximum and minimum.

5.Methodology

Sequences and Series

Definition

(a) A sequence {an} of real numbers is a function N → R defined by n 7→ an. (b) {an} is said to be non-decreasing ↑ (non-increasing ↓) if a1 ≤ a2...(a1 ≥ a2 ≥ a3..). In either case, {an} is said to be a monotone sequence. (c) {an} is said to be a bounded sequence if |an| ≤ M < ∞ for n ≥ 1. (d) A subsequence of {an} is a restriction of the function to {nk : k ≥ 1, n1 < n2 < n3...}. (e) We say that an → l ∈ R if given ε > 0, there exists N such that |an − l| < ε for n ≥ N and that an → ∞(−∞) if given M ∈ R, there exists N such that an > M(an < −M) for n ≥ N. (f) A sequence {an} is said to be a Cauchy sequence if given ε > 0, there exists N such that |an − am| < ε for n, m ≥ N. Exercises: 1. Show that (c) fails to hold if and only if there exists a subsequence of {an} converging to ±∞. 2. Show that the sequence defined by an = (1 + 1 n) n is monotone increasing and bounded. 3. Show that the sequence defined by an = 1 n + 1 n+1 + ... + 1 2n−1 is ↓ and hence convergent.

Definition. If S ⊆ R, we say that a real number M is an upper(lower) bound of S if every x ∈ S satisfies x ≤ M(x ≥ M) and M is said to be the least upper bound (lub) (greatest lower bound(glb)) if no number in (−∞, M)((M, ∞)) is an upper bound (a lower bound).

The completeness axiom Every S ⊂ R which is bounded above (below) has a lub in R.

Exercise: A sequence which is ↑ and bounded above converges to its lub.

The following results are some of the crucial consequences of the completeness of R. Monotone

Boundedness Theorem: Every bounded monotone sequence of real numbers converges to a limit in R.

Bolzano-Weierstrass Theorem: Every bounded sequence of real numbers has a monotone (and hence convergent) subsequence.

Cauchy Criterion: Every Cauchy sequence of real numbers is convergent and conversely.

Exercises: 1. Show that xn = (1 + 1 n) n is convergent. Write: xn → l. Clearly, yn = (1 + 1 n) n+1 → l, xn < yn and {yn} ↓. Hence for every n, xn < l < yn. Use a calculator and a large value of n to estimate l.

2. Let tn = 2 + 1 2! + ... + 1 n! . Show that {tn} ↑ and tn < 3. Thus {tn} is convergent. Moreover, lim xn ≥ lim tn and thus xn → P∞ n=0 1 n! . Fact: A function f : (a, b) → R is continuous if and only if whenever an → c ∈ (a, b), f(an) → f(c), provided (an), c ⊆ (a, b).

6.**Discussion**

A number *l* is called the limit of an sequence u_1, u_2, u_3,.... if for any positive number ∈>0, we can find a positive number N depending on ∈ such that u_n-*l*| <∈ for all integer n ≥**N**.

A sequence $\{u_n\}$ is said to be

bounded above if there exists a real number K such that $u_n \leq K \forall n \in N$

bounded below, if there exists a real number k such that $u_n \geq k \forall n \in N.$

bounded, if it is both bounded above and bounded below.

monotonic increasing, if $u_{n+1} \geq u_n$ *and* monotonic decreasing if

$u_{n+1} \leq u_n$

strictly increasing if $u_{n+1} > u_n$ and strictly decreasing if $u_{n+1} < u_n$.

Results to be remembered

If limit of sequence exists then, it must be unique

Every convergent sequence is bounded.

(Bolzano-Weierstrass Theorem) Every bounded sequence has a limit point

An unbounded sequence may or may not have a limit point.

A necessary and sufficient condition for the convergence of a monotonic sequence is that it is bounded.

A sequence $\{a_n\}$ is said to be Cauchy sequence if for any ε > 0, there exists a positive integer m such that $|a_n - a_m| < ε$, whenever n ≥ m.

Every Cauchy sequence is bounded but the converse may not be true.

In Cauchy sequence after a fixed positive integer say n, the difference between any two terms is less than a positive number ϵ.

(Cauchy's General Principle of Convergence)

A sequence of real numbers converges if and only if it is a Cauchy sequence.

(Cauchy's criterion)The sequence $\{a_n\}$ converges iff for every ε > 0 there exists K such that $| a_n – a_m | < ε$ whenever n, m > K.

A function f: X → Y is sequentially continuous if whenever a sequence (x_n) in X converges to a limit x, the sequence $(f(x_n))$ converges to f(x).

a function f is said to be continuous at a point c, if for any ε > 0, there exists some δ > 0, depending upon ε, such that $|f(x) – f(c)| < ε$, when $|x - c| < δ$.

A function f defined on an interval is said to be uniformly continuous in the interval I, if for each ε > 0, there exists some δ >0,depending upon ε such that $|f(x_2) – f(x_1)| < ε$, when $|x_2 – x_1| <δ$ and for all $x_2, x_1 ∈ I$.

Every uniformly continuous function on an interval is continuous on that interval, but the converse need not be true.

If $\{u_n\}$ is a sequence of real numbers, then the expression $u_1+u_2+...+u_n+....$ is called an infinite series. The infinite series u_1+ u_2+...+u_n+.... is denoted by $\sum_{n=1}^{\infty} u_n$.

A series $\sum u_n$ is said to be convergent, if the sequence $\{S_n\}$ of partial sums of $\sum u_n$ is convergent. If $\lim_{n \to \infty} S_n$ = S, then S is called the sum of the series $\sum u_n$.

The Geometric Series $1 + x + x^2 +...\infty$.

Converges if -1 < x < 1, i.e., | x | <1

diverges if x ≥ 1

Oscillates finitely if x = - 1

Oscillates infinitely if x < - 1.

7.**Analysis**

Overlooking numerical meticulousness for a second let me talk logically. For most models on the off chance that terms in the arrangement are getting more modest and more modest, at that point you can simply contemplate the digits in the halfway entireties. At the point when a digit settles down and is not, at this point affected by extra terms being added then you can with sensible sureness accept that digit is right. Obviously you have to remember adjusting, and when I state "sensible" I don't mean numerical assurance. Here and there numerical conviction isn't a choice. In such cases you might be compelled to such a heuristic thinking.

$$
\begin{aligned}
s_{10} &= 1.234544\\
s_{11} &= 1.234703\\
s_{12} &= 1.234769\\
s_{13} &= 1.234774\\
s_{14} &= 1.234770
\end{aligned}
$$

Given the information above I would bet that beyond a shadow of a doubt. On the off chance that I needed more digits I'd need to ascertain more to play it safe. That is a careful decision on my part. Obviously, I could not be right, with no extra data it is completely conceivable that the next term abuses the example. It very well may be that . This sort of irregular disparity from the example above is guaranteed by the different tests prior in this segment. By and by, we may not have an equation from which the arrangement is being produced. The arrangement could emerge out of some test estimation. We at that point simply need to take it on confidence that the example proceeds.

Frequently a numerical example is accepted despite the fact that there is no actual induction of the example. Such a models in material science are named "phenomenological". As a rule physicists are discontent with such models, one might want to clarify why a certain condition depicts a specific circumstance. One early occurrence of this was Kepler's Laws. He gave a recipe portraying the movement of planets. Notwithstanding, Kepler gave no explanation as to why this recipe should apply. One of the extraordinary victories of Newtonian mechanics was to infer Kepler's Laws as a result of Newton's Laws of movement and Newton's Universal Law of Gravitation. This story keeps on playing out today. A few researchers will discover an example, afterwards different researchers will give an explanation behind the example. At the base, all things considered, a pestering inquiry remains; for what reason is there actual law by any means? On the off chance that the universe is irregular, at that point for what reason does it have such rich and delightful actual law? There are different answers, however I accept the most consistent response to this inquiry is the undeniable one. The universe was made by an methodical being. God manufactured the universe so that not exclusively might we be able to appreciate the magnificence of the universe at any degree of detail. From our ordinary experience, to the nuclear level, to the subatomic level, it's not arbitrary, it's plan.

8.Conclusion

The inventor of chess requested the King from the Kingdom that he might be compensated in lieu of his invention with one grain of wheat for the main square of the board, two grains for the second, four grains for the third, eight grains for the fourth, etc for the 64 squares. Luckily, this clearly unobtrusive solicitation was inspected before it was conceded. By the 20th square, the prize would have added up to in excess of 1,000,000 grains of wheat; by the sixty-fourth square the number called for would have been galactic and the mass would have for surpassed all the grains in the realm. The premise of this story an arrangement of numbers that have a numerical relationship - has a large number of significant applications.

A significant number of them are past the extent of this book; however we will investigate the methods for managing various functional, and regularly engaging, issues of this sort.

References

1. Shama, G. (1998). Understanding periodicity as a process with a Gestalt structure. Educational Studies in Mathematics, 35, 255-281.
2. Hiebert, J. & Lefevre, P. (1986). Conceptual and procedural knowledge in mathematics: An introductory analysis. In J. Hiebert (Ed.), Conceptual and procedural knowledge: The case of mathematics (pp. 1-27). Hillsdale, NJ: Erlbaum.
3. Star, J. (2005). Reconceptualizing procedural knowledge. Journal for Research in Mathematics Education, 36, 404-411.

4. Oppenheim, A., Willsky, A. & Hamid, S. (1997). Signals and systems (2nd ed.). Upper Saddle River, NJ: Prentice Hall.
5. Lobato, J. (2003). How design experiments can inform a rethinking of transfer and vice versa. Educational Researcher, 32(1), 17-20.
6. Black, P., Harrison, C., Lee, C., Marshall, B., & William, D. (2003). Assessment for learning: Putting it into practice. Berkshire, England: Open University Press.
7. Miles, M. B., & Huberman, A. M. (1994). Qualitative data analysis (2nd ed.). Thousand Oaks, CA: Sage Publications