Volume-6, Issue-12, December 2019 ISSN: 2349-7637 (Online)

RESEARCH HUB – International Multidisciplinary Research Journal (RHIMRJ)

A Two-Way Generalisation of the Chow Test Under Dependent Errors

Jagabandhu Saha
Department of Economics and Politics,
Visva-Bharati University(India)

Abstract: The Chow test is not robust under dependency of errors. The presence of dependency of errors will affect level of significance as well as power of the test, especially when the sizes of the samples are small. This paper attempts to construct a test procedure where not only the equality between sets of coefficients in two linear regressions are compared, but also, in case they are not equal, detailed informations about the inequality of the sets are provided. Also, all these are accomplished for not just only two linear regressions but for all possible pairs of linear regressions out of any number of given linear regressions, resolving at the same time the problem of dependency in the errors, thus generalizing the Chow test in two directions under dependent errors. The procedure is then illustrated through comparison of growth rates of population of India for different decades, using NSSO data.

I. INTRODUCTION

It is a common practice to test the equality between sets of coefficients in two linear regressions by Chow test (Chow 1960) [1]. In the Chow test, if the null hypothesis of equality between the sets of coefficients is not rejected then there is no problem (as in the examples in his paper). But if rejected, then, naturally, one is probed to the questions: a). at which component/s the sets differ, and b). for each of those components, between the two coefficients of the two regressions concerned, which one is larger/smaller. Chow test does not provide answer to any of these questions. This problem can be resolved with some modifications of the model (Saha and Pal, 2014) [2]. Saha and Pal introduced the concept of "component wise complete comparison" (CCC)[1] in order to overcome this problem. The test procedure for CCC between every two successive regressions out of any number of given successive regressions was developed. But, however, Chow assumed independence of the regression errors. If the regression errors are dependent then the estimates may not be efficient. The presence of dependent errors will affect level of significance as well as power of the test of the regression coefficients and the test may result into wrong conclusion especially when the sizes of the samples are small. Thus if the errors are dependent then the problem of CCC aggravates and needs further modifications. Hence this paper [2] was extended (Saha and Pal, 2016) [3] in order to construct a test procedure for CCC between every two successive regressions out of any number of given successive regressions when the errors are dependent. The present paper further extends the last paper [3] in order to develop a test procedure for CCC between not only every two successive regressions out of any number of given successive regressions but between the two regressions of *all possible pairs of regressions out of any number of given regressions* when the errors are dependent, thus generalising the Chow test in two directions under dependent errors.

We may now straight go to the problem and discuss how we can arrive at a solution.

II. THE MODEL

We consider the problem of finding test procedure for CCC between the two regressions of all possible pairs of regressions out of m given regressions as follows:

[1] By complete comparison between any two parameters a and b we mean to decide whether $a < b$ or $a = b$ or $a > b$. By component wise complete comparison (CCC) between two vectors of parameters of the same size ($a_1\ a_2 \ldots a_m$) and ($b_1\ b_2 \ldots b_m$) we mean complete comparison between (a_1 and b_1), (a_2 and b_2), ... and (a_m and b_m). By CCC between/of/for two regressions with same no. of parameters we mean CCC between the two vectors of parameters of these regressions. In the paper by Saha and Pal, CCC is done between every two successive regressions out of any number of given successive regressions with same no. of parameters.

$y^{(1)} = a_1^{(1)} + a_2^{(1)} x_2^{(1)} + a_3^{(1)} x_3^{(1)} + \ldots.. + a_k^{(1)} x_k^{(1)} + u^{(1)},$

$y^{(2)} = a_1^{(2)} + a_2^{(2)} x_2^{(2)} + a_3^{(2)} x_3^{(2)} + \ldots.. + a_k^{(2)} x_k^{(2)} + u^{(2)},$

……………………………

$y^{(m)} = a_1^{(m)} + a_2^{(m)} x_2^{(m)} + a_3^{(m)} x_3^{(m)} + \ldots.. + a_k^{(m)} x_k^{(m)} + u^{(m)},$ ... (1)

where the superscripts denote the individual regressions, $n_1, n_2, \ldots, n_m$ are the nos. of observations for these regressions, the components of each of $u^{(1)}, u^{(2)}, \ldots.., u^{(m)}$ are iid $N(0, \sigma^2)$ and the errors are dependent. In order to introduce dependency of the errors we consider the Model consisting of the following assumptions:

i). $E(u^{(i)}) = 0_{nx1}$, $\forall\, i = 1, 2, \ldots\ldots, m$,

ii). $E((u^{(i)})(u^{(i)})') = \sigma^2 I_{nxn}$, $\forall\, i = 1, 2, \ldots\ldots, m$,

iii). $E((u^{(i)})(u^{(j)})') = \sigma_{ij} I_{nxn}$, $\forall\, i \neq j = 1, 2, \ldots\ldots, m$,

iv). $\sum_{mxm} = (\sigma_{ij})_{mxm}$, say, is a positive definite matrix, ... (2)

where, $\sigma_{ii} = \sigma^2$, $\forall\, i = 1, \ldots\ldots, m$, and, $n_i = n$, $\forall\, i = 1, 2, \ldots, m$

(i.e., the sample sizes for the different regressions are the same, say, n).

It is admitted that a particular type of dependency has been considered. Observe that the model is similar to that adopted in the Zellner's (1962) [4] SURE Estimation Procedure (ZSEP), and the solution here is, also, similar to that of Zellner's.

For the problem stated we proceed as follows. Firstly, for CCC between first and second regressions in (1), first and third regressions, …, first and m-th regressions, one requires to decide whether the differentials:

$c_j^{12} = a_j^2 - a_j^1 < 0$ or $= 0$ or > 0, for all $j = 1, 2, \ldots, k$,

$c_j^{13} = a_j^3 - a_j^1 < 0$ or $= 0$ or > 0, for all $j = 1, 2, \ldots, k$,

……………………

$c_j^{1m} = a_j^m - a_j^1 < 0$ or $= 0$ or > 0, for all $j = 1, 2, \ldots, k$.

Similarly, for second and third regressions, second and fourth regressions, …, second and m-th regressions, one requires to decide whether the differentials:

$c_j^{23} = a_j^3 - a_j^2 < 0$ or $= 0$ or > 0, for all $j = 1, 2, \ldots, k$,

$c_j^{24} = a_j^4 - a_j^2 < 0$ or $= 0$ or > 0, for all $j = 1, 2, \ldots, k$,

……………………

$c_j^{2m} = a_j^m - a_j^2 < 0$ or $= 0$ or > 0, for all $j = 1, 2, \ldots, k$.

……………………

Lastly, for (m-1)-th and m-th regressions, one requires to decide whether the differentials:

$c_j^{m-1,m} = a_j^m - a_j^{m-1} < 0$ or $= 0$ or > 0, for all $j = 1, 2, \ldots, k$.

A moment's reflection shows that the desired CCC, i.e., CCC for all pairs of regressions out of the m regressions in (1), will be over when all the decisions enlisted just above, in (m-1) groups, so to say, are completed. Our task is to device tests for all these decisions.

III. THE METHODOLOGY

As indicated at the end of the preceding Section, Methodology consists of (m-1) steps as ahead.

1). To build up tests in order to decide upon the differentials $c_j^{12}, c_j^{13}, \ldots, c_j^{1m}$, for all $j = 1, 2, \ldots, k$:

Firstly, we combine the m regressions in (1) into a single regression equation model as follows.

$$\begin{pmatrix} y_1^{(1)} \\ \vdots \\ y_n^{(1)} \\ y_1^{(2)} \\ \vdots \\ y_n^{(2)} \\ \vdots \\ y_1^{(m)} \\ \vdots \\ y_n^{(m)} \end{pmatrix} = \begin{pmatrix} 1 & x_{21}^{(1)} & \cdots & x_{k1}^{(1)} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{2,n}^{(1)} & \cdots & x_{k,n}^{(1)} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 1 & x_{21}^{(2)} & \cdots & x_{k1}^{(2)} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 1 & x_{2,n}^{(2)} & \cdots & x_{k,n}^{(2)} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{21}^{(m)} & \cdots & x_{k1}^{(m)} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{2,n}^{(m)} & \cdots & x_{k,n}^{(m)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ \vdots \\ a_k^{(1)} \\ a_1^{(2)} \\ \vdots \\ a_k^{(2)} \\ \vdots \\ a_k^{(m)} \\ \vdots \\ a_k^{(m)} \end{pmatrix} + \begin{pmatrix} u_1^{(1)} \\ \vdots \\ u_n^{(1)} \\ u_1^{(2)} \\ \vdots \\ u_n^{(2)} \\ \vdots \\ u_1^{(m)} \\ \vdots \\ u_n^{(m)} \end{pmatrix} \quad \ldots(3)$$

The solution for the single equation model is same as that of finding solution separately for each equation in (1). The benefit of writing a single equation model is that the error terms in (1) are now constituents of the one error vector of this single equation model and hence we can now conceive the dependency of these error terms easily, dependency in the sense as formulated by (2). In addition to introducing dependency of the error terms we also want to decide upon the differentials c_j^{12}, c_j^{13}, ..., c_j^{1m}, for all j = 1, 2, …, k. For that we slightly change the model further as follows.

$$\begin{pmatrix} y_1^{(1)} \\ \vdots \\ y_n^{(1)} \\ y_1^{(2)} \\ \vdots \\ y_n^{(2)} \\ \vdots \\ y_1^{(m)} \\ \vdots \\ y_n^{(m)} \end{pmatrix} = \begin{pmatrix} 1 & x_{21}^{(1)} & \cdots & x_{k1}^{(1)} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{2,n}^{(1)} & \cdots & x_{k,n}^{(1)} & 0 & 0 & \cdots & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 1 & x_{21}^{(2)} & \cdots & x_{k1}^{(2)} & 1 & x_{21}^{(2)} & \cdots & x_{k1}^{(2)} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{2,n}^{(2)} & \cdots & x_{k,n}^{(2)} & 1 & x_{2,n}^{(2)} & \cdots & x_{k,n}^{(2)} & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{21}^{(m)} & \cdots & x_{k1}^{(m)} & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{21}^{(m)} & \cdots & x_{k1}^{(m)} \\ \vdots & \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{2,n}^{(m)} & \cdots & x_{k,n}^{(m)} & 0 & 0 & \cdots & 0 & \cdots & 1 & x_{2,n}^{(m)} & \cdots & x_{k,n}^{(m)} \end{pmatrix} \begin{pmatrix} a_1^{(1)} \\ \vdots \\ a_k^{(1)} \\ c_1^{12} \\ \vdots \\ c_k^{12} \\ \vdots \\ c_1^{1m} \\ \vdots \\ c_k^{1m} \end{pmatrix} + \begin{pmatrix} u_1^{(1)} \\ \vdots \\ u_n^{(1)} \\ u_1^{(2)} \\ \vdots \\ u_n^{(2)} \\ \vdots \\ u_1^{(m)} \\ \vdots \\ u_n^{(m)} \end{pmatrix} \quad \ldots(4)$$

Let us, for convenience, rewrite (4) as:

$$Y = X\,c + U, \qquad \ldots..(5)$$

where, Y_{Nx1} = the Y-vector in (4), X_{NxK} = the X-matrix in (4), c_{Kx1} = the coefficient-vector in (4) and U_{Nx1} = the disturbance-vector in (4), N = nm and K = km.

We can now run regression with (5), estimate c and perform tests in order to decide upon the differentials c_j^{12}, c_j^{13}, ..., c_j^{1m}, for all j = 1, 2, …, k. But model (5) is a Generalised Least Squares Model (GLSM) [5]. The estimation procedure will depend on the variance- covariance matrix of the regression error, which is given as:

$$(D(U))_{NxN} = V, \text{ say, } = \begin{pmatrix} \sigma^2 I_{nxn} & \sigma_{12} I_{nxn} & \cdots & \cdots & \cdots & \sigma_{1m} I_{nxn} \\ \sigma_{21} I_{nxn} & \sigma^2 I_{nxn} & \cdots & \cdots & \cdots & \sigma_{2m} I_{nxn} \\ \vdots & & & & & \\ \vdots & & & & & \\ \vdots & & & & & \\ \sigma_{m1} I_{nxn} & \sigma_{m2} I_{nxn} & \cdots & \cdots & \cdots & \sigma^2 I_{nxn} \end{pmatrix},$$

or, $V = \sum_{mxm} \otimes I_{nxn}$, where, … (6)

$$\Sigma_{mxm} = \begin{pmatrix} \sigma^2 & \sigma_{12} & \dots & \dots & \sigma_{1m} \\ \sigma_{21} & \sigma^2 & \dots & \dots & \sigma_{2m} \\ \dots & & & & \\ \dots & & & & \\ \sigma_{m1} & \sigma_{m2} & & & \sigma^2 \end{pmatrix}.$$

The GLS estimator of c based on (5) is given as:

$$c^* = (X'V^{-1}X)^{-1}X'V^{-1}Y, \qquad \text{...... (7)}$$

and its dispersion matrix is:

$$D(c^*) = (X'V^{-1}X)^{-1}. \qquad \text{...... (8)}$$

But due to (6), V is unknown as Σ_{mxm} is so. So it is not possible to use (7) and (8) in practice, particularly for the testing purposes we are aimed at. Henceforth let us proceed following Zellner [4] .

Firstly, we need to estimate V. For that we need to estimate Σ_{mxm} and that is done as follows. The steps are:

i). Apply OLS separately to each of the regressions in (1); let the residual vector for the i-th regression be denoted as e^i, for all i = 1, 2, …, m,

ii). Estimate σ^2 as: $s^2 = [\, (e^{1\prime}e^1) + \dots + (e^{m\prime}e^m)\,]/(m(n-k))$.

iii). Estimate σ_{ij} as: $s_{ij} = (e^{i\prime}e^j)/(n-k), \ \forall\ i \neq j = 1, 2, \dots, m$.

Then, estimated $\Sigma_{mxm,}$ say, S_{mxm}, is: $S_{mxm} = (s_{ij})_{mxm}$, where $s_{ii} = s^2$, for all i = 1, 2, ..., m. Then, V is estimated as:

$$\hat{V} = S_{mxm} \otimes I_{nxn}. \qquad \text{…… (9)}$$

Now we replace V in (7) by $\hat{V}$ as given by (9) and form the estimator:

$$c^{**} = (X'(\hat{V})^{-1}X)^{-1}X'(\hat{V})^{-1}Y. \qquad \text{...... (10)}$$

Then it follows that $(n^{1/2})$ (c** - c) has asymptotic normal distribution and the dispersion matrix of c** is:

$$D(c^{**}) = (X'(\hat{V})^{-1}X)^{-1} + {}_0(n^{-1}),$$

where ${}_0(n^{-1})$ denotes terms of high order of smallness than n^{-1}.

So, for large value of n, c** is normally distributed. Also, evidently, for large n, ${}_0(n^{-1})$ is negligible and then,

$$D(c^{**}) \simeq (X'(\hat{V})^{-1}X)^{-1}.$$

So, for large n, we have:

$$c^{**} \sim N_K(\, c_{Kx1}\, ,\ (X'(\hat{V})^{-1}X)^{-1}). \qquad \text{..... (11)}$$

Now, the tests that we require are obvious, provided that n is sufficiently large which we assume for rest of the paper. Representing,

$$c^{**} = (c^{**}_1\ c^{**}_2\ c^{**}_3\ \dots\dots\ c^{**}_K)' ,$$

$c = (c_1 \quad c_2 \quad c_3 \quad \ldots\ldots \quad c_K)'$, and

$(X'(\hat{V})^{-1}X)^{-1} = (a_{ij})_{KxK}$,

we have: $(c^{**}_i - c_i) / (a_{ii})^{1/2} \sim N(0, 1)$, for all $i = 1, 2, 3, \ldots\ldots, K$.

Hence for the null hypothesis: $H_0 : c_i = 0$,

the test statistic is: $T = c^{**}_i / (a_{ii})^{1/2}$, and ….. (12)

$T \sim N(0, 1)$, under H_0, for all $i = 1, 2, 3, \ldots\ldots, K$.

Needless to say that each of the tests here is a normal test.

This completes tests for CCC for (m-1) pairs of regressions: first and second, first and third, ..., first and m-th.

2). To build up tests in order to decide upon the differentials $c_j^{23}, c_j^{24}, \ldots, c_j^{2m}$, for all j = 1, 2, …, k:

The entire procedure here is exactly similar as in the step 1). but with the last (m-1) regressions in (1) instead of all the m regressions in (1).

This completes tests for CCC for (m-2) pairs of regressions: second and third, second and fourth, ..., second and m-th.

……………………

m-1). Lastly, to build up tests in order to decide upon the differentials $c_j^{m-1,m}$, for all j = 1, 2, …, k.

Here also the entire procedure is exactly similar as in the step 1). but with only the last two Regressions in (1) ((m-1)th and mth).

This completes tests for CCC for one and the last pair of regressions: (m-1)th and mth.

Thus, tests for CCC for all possible pairs of regressions out of the m given regressions in (1) is complete.

IV. ILLUSTRATION

In the context of rate of growth of population in India, we consider three regression equations (m = 3) as follows. With state level population of India, we first define the following four variables:

X_1 = size of the population in a state of India in 1981,

X_2 = size of the population in a state of India in 1991,

X_3 = size of the population in a state of India in 2001,

X_4 = size of the population in a state of India in 2011,

the sources of these data being Census of India (1981, 1991, 2001, 2011) [6].

Let us now define variables Y_1, Y_2, Y_3 as follows:

$Y_1 = X_2 - X_1$ (i.e., growth/increase of population during: 1981 to 1991)Y

$Y_2 = X_3 - X_2$ (i.e., growth/increase of population during: 1991 to 2001)Y

$Y_3 = X_4 - X_3$ (i.e., growth/increase of population during: 2001 to 2011).

We now consider three regressions as follows:

$$\left.\begin{array}{l} Y_1 = \beta_1 X_1 + U_1 \\ Y_2 = \beta_2 X_2 + U_2 \\ Y_3 = \beta_3 X_3 + U_3 \end{array}\right] \quad \ldots (13)$$

β_1, β_2 and β_3 are nothing but the rates of growth of population over the decades: 1981 to 1991, 1991 to 2001 and 2001 to 2011 respectively (to be referred as first decade, second decade and so on).

Following the last Section, for CCC of all possible pairs out of the three regressions in (13), we have here two steps ((m-1) steps with m = 3) as follows.

1). First consider tests for CCC of two pairs of regressions: first and second, first and third.

Again, following the last Section, we apply OLS separately to each of the above three regressions in (14). It may be noted that each of these regressions is a regression without an intercept term. (The no. of observations for each regression here is n = 32(no. of States in India); so, we have here: n = 32, k = 1, m = 3.)

The Residual vectors of these three regressions are first obtained. It is now a routine calculation to get the sum of squares and the sum of products of the residual vectors and hence the estimates of σ^2 and σ_{ij} (s^2 and s_{ij}) using the formula as given already. We then use the following steps to get the value of c** in (10) and D(c**) in (11), the value of c** representing as: first component gives the estimate of the growth rate in the first decade, and the second and the third components give respectively the estimates of changes in the growth rates over first decade to second decade and over first decade to third one.

1. Construct the matrix S_{3x3} and compute $(S_{3x3})^{-1}$.

2. Compute $(\hat{V})^{-1}_{96x96} = (S_{3x3})^{-1} \otimes I_{32x32}$.

3. Compute c** as: $c^{**} = (X'(\hat{V})^{-1} X)^{-1} X'(\hat{V})^{-1} Y$ and its dispersion matrix, D(c**), as:

$D(c^{**}) = (X'(\hat{V})^{-1} X)^{-1}$.

The estimate of growth rate in the first decade is 0.231 and the estimates of the changes concerned are respectively -0.107 and -0.050 with the corresponding T-values, given by (12), as 10.475, -3.857 and -1.794 respectively. Compared with table-value, it may be concluded that the second T-value indicates that there is a decline in growth rate as one moves from the first decade to the second one and the third T-value indicates that the change in growth rate as one moves from the first decade to the third one is insignificant [2]. Hence it may be concluded that the first regression differs significantly from the second one but not from the third one.

2). Next and lastly we consider test for CCC of one pair of regressions: second and third.

We consider now the last two regressions in (14) and proceed exactly in the similar way as in the step 1). with these two regressions only. The results come out as follows.

The estimate of growth rate in the second decade is 0.124 and the estimate of the change concerned, i.e., the estimate of the change in growth rate as one moves from second decade to the third one, is 0.057 with the corresponding T-values, given by (12), as 6.987 and 2.412 respectively. Compared with table-value, it may be concluded that the second T-value indicates that there is a rise in growth rate as one moves from the second decade to the third one. Hence it may be concluded that the second regression differs from the third one significantly. This is quite expected from the behavior of the first regression.

This evidently completes tests for CCC for all possible pairs of regressions out of the three given regressions in (13).

[2] All tests throughout this Illustration is carried out at 5% level of significance.

V. CONCLUSION

The test procedure suggested above enables one to perform CCC between the two regressions in all possible pairs of regressions out of several given regressions when the errors are dependent, generalising Chow test in two directions under dependent errors. This, again, has at least two important implications as follows.

Suppose the regressions are arranged successively in a definite order. Then, evidently, this procedure also enables one to perform CCC between every two successive regressions out of all the given regressions. Again, one of the important implications of this is as follows. Suppose each one of the given regressions pertains to a time period/point and the regressions are arranged in increasing order of time and the investigator is in search of a) existence of structural breakthrough and b) detection of the point/s (here, by a point we mean a time period or a time point) where it occurs, if there is any such at all. Not only the point/s of structural breakthrough, if there is any at all, through our procedure we get something more. For every such point we get CCC of the vectors of coefficients of the two regressions associated with that point. Actually, it is not necessary that the regressions need to be ordered in increasing/decreasing order of time; it is sufficient for the regressions to be ordered in a well defined sense, e.g., (i) in order of space, e.g., regressions pertain to some states of India arranged from North to South, (ii) in increasing order of income, e.g., regressions pertain to some groups of peoples arranged in increasing order of income, etc. It seems that the concept of "Structural Change" can be extended, not pertaining to only "order of time" but pertaining to 'any well defined order' in which the regressions can be meaningfully arranged.

Since by this procedure one can perform CCC between the two regressions in all pairs of regressions out of the given regressions, one can partition the set of the given regressions into, say, clusters, every cluster consisting of those regressions for which it is true that the vectors of coefficients of any two of these regressions do not differ significantly. As an example, we can consider all the States of India in a particular year and for each State consider regression of consumption on income (or an appropriate substitute of it) based on observations on the peoples of that State for that year and then perform CCC of all these regressions and then cluster the States, each cluster including those States which has same income-consumption pattern in an appropriate sense.

REFERENCES

1. Chow, Gregory C. (1960): Tests of Equality between Sets of Coefficients in Two Linear Regressions, Econometrica, Vol. 28, No. 3, pp. 591-605.
2. Saha, J. and Pal, M. (2014): A Modified Chow Test Approach Towards Testing Differences in The Engel Elasticities, Asian-African Journal of Economics and Econometrics, Vol.14, No.1, 2014; 57-67.
3. Saha, J. And Pal, M. (2016): Test for Structural Change under Dependent Errors: The Case of Successive Regressions, International Journal of Applied Business and Economic Research, Vol. 14, No. 12, (2016): 8657-8666.
4. Zellner, Arnolds (1962): An Efficient Method of Estimating Seemingly Unrelated Regressions and Tests for Aggregation Bias, Journal of the American Statistical Association, Vol.57, 1962, pp.348-368.
5. Johnston J. (1984): Econometric Methods, Third Edn., McGraw-Hill, New York.
6. Census of India (1981, 1991, 2001, 2011): Government of India.