Volume-2, Issue-2, February-2015 **ISSN: 2349-7637 (Online)**

RESEARCH HUB – International Multidisciplinary Research Journal (RHIMRJ)

Research Paper

Available online at: www.rhimrj.com

Order Level Lot Size Inventory Model for Detererioratting Items Under Quadratic Demand with Time Dependent IHC and Partial Backlogging

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***Abstract:** In this paper, we have analysed a deterministic inventory model for deteriorating items with time-dependent quadratic demand and holding cost is time-dependent. Two parameter Weibull distributions are used to represent the distribution of time to deterioration. In the model considered here, shortages are allowed and partially backlogged. The backlogging rate is assumed to be dependent on the length of waiting for the next replenishment. The longer the waiting time is, the smaller the backlogging rate would be.The model is solved analytically to obtain the optimal solution of the problem. The derived model is illustrated by a numerical example and its sensitivity analysis is carried out.*

***Keywords:** Inventory, quadratic demand, Weibull distribution, deteriorating items, time-varying holding cost, shortages, partial backlogging.*

I. INTRODUCTION

Inventory system is one of the main streams of the Operation Research which is essential in business enterprises and industries.Inventory may be considered as an accumulation of a product that would be used to satisfy future demands for that product. It needs scientific way of exercising inventory model.

The pioneering work of Harris [8] in inventory models istreated by mathematical techniques. He developed the simplest inventory model, the Economic Order Quantity (EOQ) model which was later popularized by Wilson [25]. Relaxation of some assumptions in the formulation of the EOQ model led to the development of other inventory models that effectively tackle several other inventory problems occurring in day-to-day life.

An optimal replenishment policy is dependent on ordering cost, inventory carrying cost and shortage cost. An important problem confronting a supply manager in any modern organization is the control and maintenance of inventories of deteriorating items. Fortunately, the rate of deterioration is too small for items like steel, toys, glassware, hardware, etc. There is little requirement for considering deterioration in the determination of economic lot size.

Inventory of deteriorating items first studied by Whitin [24]. He considered the deterioration of fashion goods at the end of prescribed storage period. Ghare and Schrader [6] extended the classical EOQ formula to include exponential decay, wherein a constant fraction of on hand inventory is assumed to be lost due to deterioration.Covert and Philip [3] and Shah and Jaiswal [19] carried out an extension to the above model by considering deterioration of Weibull and general distributions respectively. Dave and Patel [4] first developedan inventory model for deteriorating items with time proportional demand, instantaneous replenishment and no shortagesallowed. Many researchers such as Park [16] and Hollier and Mak [9] also considered constant backlogging rates in their inventory models. Nahmias [13] gave a review on perishable inventory theory. Rafaat [17] described survey of literature on continuously deteriorating inventory model. He focused to present an up-to-date and complete review of the literature for the continuously deteriorating mathematical inventory models.

All researchers assume that during shortage period all demand either backlogged or lost. In reality, it is observed that some customers are willing to wait for the next replenishment. Abad [1] considered this phenomenon in his model, optimal pricing and lot sizing under conditions of perishable and partial backordering. He assumes that the backlogging rate depends upon the waiting time for the next replenishment. But he does not include the stock out cost (back order cost and lost sale cost).

Backlogging rate is assumed to be a fixed fraction of demand rate during the shortage period. However, in some inventory system, for many stocks such as fashionable commodities, the length of the waiting time for the next replenishment becomes a major factor for determining whether the backlogging will be accepted or not. Chang and Dye [2] developed an inventory model with time varying demand and partial backlogging. He considered that if longer the waiting time smaller the backlogging rate would be.Therefore, the backlogging rate is variable and is dependent on the waiting time for the next replenishment. The proportion of the customer who would like to accept backlogging at time t is decreasing with the waiting time for the next replenishment. So to take care for this situation he defined a backlogging rate s. t. where ti is the time at which the ith replenishment is making and α is backlogging parameter.

$$B(t)=\frac{1}{1+\alpha(t_i-1)}$$

Goyal and Giri [7] gave recent trends of modeling in deteriorating inventory. Ouyang, Wu and Cheng [15] developed an inventory model for deteriorating items with exponential declining demand and partial backlogging. Dye and Ouyang [5] found an optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging. They assume that a fraction of customers who backlog their orders increases exponentially as the waiting time for the next replenishment decreases. Singh and Singh [20] presented an EOQ inventory model with Weibull distribution deterioration, Ramp type demand and Partial Backlogging. NitaShah and Kunal Shukla [14] developed a deteriorating inventory model for waiting time partial backlogging when demand is constant and deterioration rate is constant.Singh, T.J., Singh, S.R. and Dutt, R. [21] extended an EOQ model for perishable items with power demand and partial backlogging.Skouri, Konstantaras, Papachristos and Ganas [22] developed an Inventory models with ramp type demand rate, partial backlogging and Weibell's deterioration rate.

An exponentially time-varying demand also seems to be unrealistic because an exponential rate of change is very high and it is doubtful whether the market demand of any product may undergo such a high rate of change as exponential.

In reality, the demand and holding cost for physical goods may be time dependent. Time also plays and important role in the inventory system. So, in this paper we consider that demand and holding cost are time dependent.

Recently, Mishra and Singh [12] developed a deteriorating inventory model with partial backlogging when demand and deterioration rate is constant. Vinodkumar Mishra [23] developed an inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost. J. Jagadeeswari and P. K. Chenniappan [10] developed an order level inventory model for deteriorating items with time – quadratic demand and partial backlogging. Sarala Pareek and Garima Sharma [18] developed an inventory model with Weibull distribution deteriorating item with exponential declining demand and partial backlogging.

Recently, Kirtan Parmar and U. B. Gothi [11] developed a deterministic inventory model for deteriorating items where time to deterioration has Exponential distribution.In this model, shortages are not allowed and holding cost is time-dependent. Here, we have extended above deterministic inventory model by taking two parameter Weibull distributions to represent the distribution of time to deterioration and shortages are allowed and partially backlogged.

II. NOTATIONS

The mathematical model in this paper is developed using the following notations:

1. Q(t) : The instantaneous state of the inventory level at any time t. $(0 \le t \le T)$
2. R(t) : Quadratic demand rate.
3. A : Ordering cost per order.
4. C_h : Inventory holding cost per unit per unit time.
5. C_d : Deterioration cost per unit per unit time.
6. C_s : Shortage cost due to lost sales per unit.
7. Q : Order quantity in one cycle.
8. p_c : Purchase cost per unit.
9. l : Opportunity cost due to lost sales per unit.
10. t_1 : The time at which the inventory level reaches zero (decision variable) $(t_1 \ge 0)$
11. T : The length of cycle time (decision variable).
12. IM : The maximum inventory level during [0, T].
13. IB : The maximum inventory level during shortage period.
14. $TC(t_1,T)$: Total cost per unit time.

III. ASSUMPTIONS

The model is derived under the following assumptions.

1. The inventory system deals with single item.
2. The annual demand rate is a quadratic function of time and it is $R(t) = a+bt+ct^2$ $(a, b, c > 0)$
3. Holding cost is linear function of time and it is $C_h = h + rt$ $(h, r > 0)$
4. The lead time is zero.
5. Time horizon is finite.
6. No repair or replacement of the deteriorated items takes place during a given cycle.
7. Total inventory cost is a real, continuous function which is convex to the origin.
8. Shortages are allowed and partially backlogged.

During stock out period, the backlogging rate is variable and is dependent on the length of the waiting time for the next replenishment. The backlogging rate is assumed to be $\frac{1}{1+\delta\,(T-t)}$ where the backlogging parameter δ (0< δ<1) is a positive constant and (T – t) is waiting time ($t_1 \le t \le T$).

IV. Mathematical Model and Analysis

Here, the replenishment policy of a deteriorating item with partial backlogging is considered. The objective of the inventory problem is to determine the optimal order quantity and the length of ordering cycle so as to keep the total relevant cost as low as possible. The behavior of inventory system at any time is shown in figure 1.

Replenishment is made at time t = 0 and the inventory level is at its maximum level S. Due to both the market demand and deterioration of the item, the inventory level decreases during the period [0, t_1], and ultimately falls to zero at t = t_1. Thereafter, shortages are allowed to occur during the time interval [t_1, T], and all of the demand during the period [t_1, T] is partially backlogged.

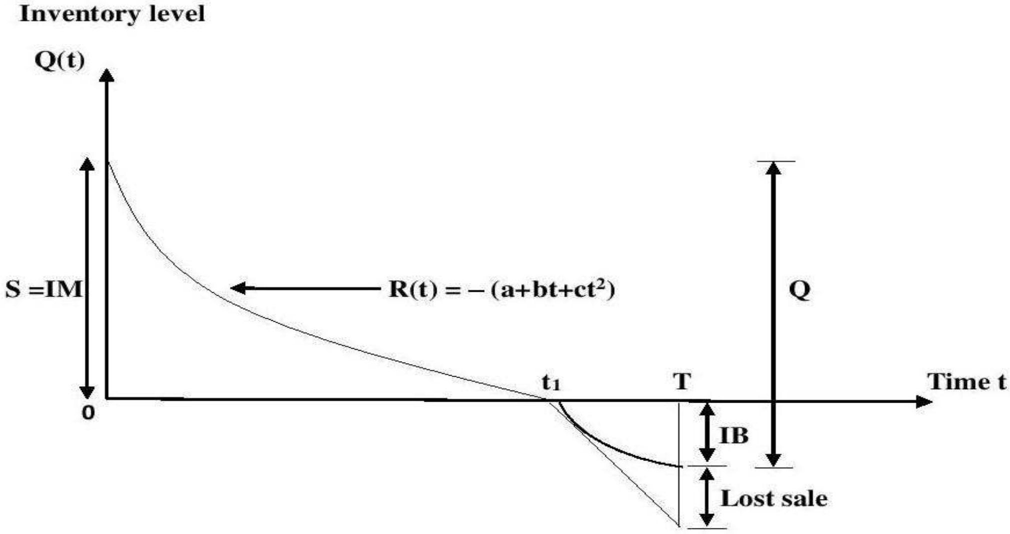

Figure 1: Graphical representation of the inventory system

As described above, the inventory level decreases owing to demand rate as well as deterioration during inventory interval [0, t_1]. Hence, the differential equation representing the inventory status is given by

$$\frac{dQ(t)}{dt} + \alpha\beta t^{\beta-1} Q(t) = -\left(a + bt + ct^2\right), \qquad (0 \le t \le t_1) \tag{1.1}$$

During the shortage interval [t_1,T], the demand at time t is partly backlogged at the fraction $\frac{1}{1+\delta\,(T-t)}$ Thus, the differential equation governing the amount of demand backlogged is as below.

$$\frac{dQ(t)}{dt} = -\frac{\left(a + bt + ct^2\right)}{1+\delta\left(T-t\right)}, \qquad (t_1 \le t \le T) \tag{1.2}$$

The boundary conditions are Q(0) = IM = S, Q(t_1) = 0 and Q(T) = 0.
Equation (1.1) is linear differential equation and its solution is given by

$$e^{\alpha t^{\beta}} Q(t) = -\int (a + bt + ct^2) e^{\alpha t^{\beta}}\, dt$$
$$= -\int (a + bt + ct^2)(1 + \alpha t^{\beta})\, dt \qquad \text{(Neglecting higher powers of } \alpha)$$

Using the boundary condition Q(t_1) = S, we get k = S.

$$Q(t) = \left\{ S - \left[at + \frac{bt^2}{2} + \frac{ct^3}{3} + a\alpha\frac{t^{\beta+1}}{\beta+1} + \alpha b\frac{t^{\beta+2}}{\beta+2} + \alpha c\frac{t^{\beta+3}}{\beta+3} \right] \right\} e^{\alpha t^{\beta}} \tag{1.3}$$

$$= \left\{ \begin{array}{l} S\left(1-\alpha t^{\beta}\right)-\left(at+\dfrac{bt^2}{2}+\dfrac{ct^3}{3}+a\alpha\dfrac{t^{\beta+1}}{\beta+1}+\alpha b\dfrac{t^{\beta+2}}{\beta+2}+\alpha c\dfrac{t^{\beta+3}}{\beta+3}\right) \\ +\left(a\alpha t^{\beta+1}+\alpha b\dfrac{t^{\beta+2}}{2}+\dfrac{\alpha c t^{\beta+3}}{3}+a\alpha^2\dfrac{t^{2\beta+1}}{\beta+1}+\alpha^2 b\dfrac{t^{2\beta+2}}{\beta+2}+\alpha^2 c\dfrac{t^{2\beta+3}}{\beta+3}\right) \end{array} \right\}$$

(neglecting higher powers of α) (1.4)

With the boundary condition $Q(t_1) = 0$, the solution of equation (1.3) is

$$S = IM = at_1 + \frac{bt_1^2}{2} + \frac{ct_1^3}{3} + a\alpha\frac{t_1^{\beta+1}}{\beta+1} + \alpha b\frac{t_1^{\beta+2}}{\beta+2} + \alpha c\frac{t_1^{\beta+3}}{\beta+3} \tag{1.5}$$

and

$$\Rightarrow Q(t) = \left(\begin{array}{l} at_1 + \dfrac{bt_1^2}{2} + \dfrac{ct_1^3}{3} + a\alpha\dfrac{t_1^{\beta+1}}{\beta+1} + \alpha b\dfrac{t_1^{\beta+2}}{\beta+2} + \alpha c\dfrac{t_1^{\beta+3}}{\beta+3} - at - \dfrac{bt^2}{2} - \dfrac{ct^3}{3} \\ -\left(a\alpha t_1 + \alpha b\dfrac{t_1^2}{2} + \alpha c\dfrac{t_1^3}{3}\right)t^{\beta} + a\alpha\beta\dfrac{t^{\beta+1}}{\beta+1} + \alpha b\beta\dfrac{t^{\beta+2}}{2(\beta+2)} + \alpha\beta c\dfrac{t^{\beta+3}}{3(\beta+3)} \end{array} \right)$$

$$(0 \le t \le t_1) \tag{1.6}$$

Equation (1.2) is linear differential equation and its solution is given by

$$Q(t) = -\int\left[\frac{(a+bt+ct^2)}{1+\delta\,(T-t)}\right]dt$$

With the boundary condition $Q(t_1) = 0$, we get

$$\Rightarrow Q(t) = \left\{\left(\frac{b\delta + c\delta T + c}{\delta^2}\right)(t - t_1) + \left(\frac{c}{2\delta}\right)\left(t^2 - t_1^2\right) + \xi \ln\left[\frac{1+\delta(T-t)}{1+\delta(T-t_1)}\right]\right\}$$

$$\left(\text{where } \xi = \frac{a+bT+cT^2}{\delta} + \frac{b+2cT}{\delta^2} + \frac{c}{\delta^3}\right) \quad (t_1 \le t \le T) \tag{1.7}$$

The total cost per time unit comprises of the following costs

1. **The Ordering Cost**

   OC = A (1.8)

2. **The deterioration cost during the period [0, t_1]**

$$DC = C_d\{\, S - \int_0^{t_1} R(t)dt \,\}$$

$$\Rightarrow DC = C_d\left(a\alpha\frac{t_1^{\beta+1}}{\beta+1} + \alpha b\frac{t_1^{\beta+2}}{\beta+2} + \alpha c\frac{t_1^{\beta+3}}{\beta+3}\right) \tag{1.9}$$

3. **The inventory holding cost during the period [0, t_1]**

$$IHC = \int_0^{t_1} C_h \cdot Q(t)dt$$

$$\Rightarrow IHC=\left\{\begin{array}{l} ah\frac{t_1^2}{2}+(ar+2bh)\frac{t_1^3}{6}+(br+2ch)\frac{t_1^4}{8}+cr\frac{t_1^5}{10}+\frac{(a\alpha\beta h)t_1^{\beta+2}}{(\beta+1)(\beta+2)}+\frac{\alpha\beta(ar+a\beta r+4bh+2b\beta h)t_1^{\beta+3}}{2(\beta+1)(\beta+2)(\beta+3)} \\ +\frac{\alpha\beta(br+b\beta r+4ch+2\beta ch)t_1^{\beta+4}}{2(\beta+1)(\beta+2)(\beta+4)}+\frac{(\alpha\beta cr)t_1^{\beta+5}}{2(\beta+2)(\beta+5)} \end{array}\right\} \tag{1.10}$$

4. The shortage cost per cycle

$$SC=-C_s\int_{t_1}^{T}Q(t)dt$$

$$=-C_s\left\{\int_{t_1}^{T}\left[\left(\frac{b\delta+c\delta T+c}{\delta^2}\right)(t-t_1)+\left(\frac{c}{2\delta}\right)(t^2-t_1^2)+\xi\ln\left(\frac{1+\delta(T-t)}{1+\delta(T-t_1)}\right)\right]dt\right\}$$

$$\Rightarrow SC=C_s\left\{-\frac{(3b\delta+4c\delta T+2c\delta t_1+3c)(T-t_1)^2}{6\delta^2}+\xi(T-t_1)-\frac{\xi}{\delta}\ln\left[1+\delta(T-t_1)\right]\right\} \tag{1.11}$$

5. Lost sales cost per cycle

$$LSC=\ell\left\{\int_{t_1}^{T}\left[1-\frac{1}{1+\delta(T-t)}\right](a+bt+ct^2)dt\right\}$$

$$\Rightarrow LSC=\ell\left\{\left(\frac{a\delta^2+b\delta+c\delta T+c}{\delta^2}\right)(T-t_1)+\left(\frac{b\delta+c}{2\delta}\right)(T^2-t_1^2)+\frac{c}{3}(T^3-t_1^3)-\xi\ln(1+\delta(T-t_1)\right\} \tag{1.12}$$

The maximum backordered inventory is obtained at t = T and it is denoted by IB. Then from equation (1.7)

$$IB=-Q(T)$$

$$IB=-\left(\frac{b\delta+c\delta T+c}{\delta^2}\right)(T-t_1)-\left(\frac{c}{2\delta}\right)(T^2-t_1^2)+\xi\{\ln[1+\delta(T-t_1)]\} \tag{1.13}$$

Thus, the order size during total interval [0, T] is given by

$$Q=IM+IB$$

6. Purchase cost per cycle

$$PC=p_cQ$$

$$=p_c\left\{\begin{array}{l} at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}+a\alpha\frac{t_1^{\beta+1}}{\beta+1}+\alpha b\frac{t_1^{\beta+2}}{\beta+2}+\alpha c\frac{t_1^{\beta+3}}{\beta+3}-\left(\frac{b\delta+c\delta T+c}{\delta^2}\right)(T-t_1) \\ -\left(\frac{c}{2\delta}\right)(T^2-t_1^2)+\xi\ln\left[1+\delta(T-t_1)\right] \end{array}\right\} \tag{1.14}$$

Hence the total cost per unit time is given by

$$TC(t_1,T)=\frac{1}{T}(OC+DC+IHC+SC+LSC+PC)$$

$$
=\frac{1}{T}\left\{\begin{aligned}
&A+C_d\left(a\alpha\frac{t_1^{\beta+1}}{\beta+1}+\alpha b\frac{t_1^{\beta+2}}{\beta+2}+\alpha c\frac{t_1^{\beta+3}}{\beta+3}\right)\\
&+\left\{\begin{aligned}&(ah)\frac{t_1^2}{2}+(ar+2bh)\frac{t_1^3}{6}+(br+2ch)\frac{t_1^4}{8}+(cr)\frac{t_1^5}{10}+\frac{(a\alpha\beta h)\,t_1^{\beta+2}}{(\beta+1)(\beta+2)}\\
&+\frac{\alpha\beta(ar+a\beta r+4bh+2b\beta h)\,t_1^{\beta+3}}{2(\beta+1)(\beta+2)(\beta+3)}+\frac{\alpha\beta(br+b\beta r+4ch+2\beta ch)\,t_1^{\beta+4}}{2(\beta+1)(\beta+2)(\beta+4)}+\frac{(\alpha\beta cr)\,t_1^{\beta+5}}{2(\beta+2)(\beta+5)}\end{aligned}\right\}\\
&+C_s\left\{-\frac{(3b\delta+4c\delta T+2c\delta t_1+3c)(T-t_1)^2}{6\delta^2}+\xi(T-t_1)-\frac{\xi}{\delta}\ln[1+\delta(T-t_1)]\right\}\\
&+\ell\left\{\left(\frac{a\delta^2+b\delta+c\delta T+c}{\delta^2}\right)(T-t_1)+\left(\frac{b\delta+c}{2\delta}\right)(T^2-t_1^2)+\frac{c}{3}(T^3-t_1^3)-\xi\ln(1+\delta(T-t_1))\right\}\\
&+p_c\left\{\begin{aligned}&at_1+\frac{bt_1^2}{2}+\frac{ct_1^3}{3}+a\alpha\frac{t_1^{\beta+1}}{\beta+1}+\alpha b\frac{t_1^{\beta+2}}{\beta+2}+\alpha c\frac{t_1^{\beta+3}}{\beta+3}-\left(\frac{b\delta+c\delta T+c}{\delta^2}\right)(T-t_1)\\
&-\left(\frac{c}{2\delta}\right)(T^2-t_1^2)+\xi\ln[1+\delta(T-t_1)]\end{aligned}\right\}
\end{aligned}\right\} \tag{1.15}
$$

Our objective is to determine optimum value of t_1 and T so that $TC(t_1, T)$ is minimum. The values of t_1 and T, for which the total cost $TC(t_1, T)$is minimum, is the solution of equations $\frac{\partial TC(t_1,T)}{\partial t_1}=0$ and $\frac{\partial TC(t_1,T)}{\partial T}=0$ satisfying the condition

$$\left\{\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1^2}\right)\left(\frac{\partial^2 TC(t_1,T)}{\partial T^2}\right)-\left(\frac{\partial^2 TC(t_1,T)}{\partial t_1 \partial T}\right)^2\right\}>0$$

The optimal solution of the equations (1.15) can be obtained by using appropriate software. This has been illustrated by the following numerical example.

V. NUMERICAL EXAMPLE

The above model is illustrated by the following numerical illustration. We consider the following parametric values for a = 20, b = 15, c = 10, Cs = 2, ℓ = 15, p_c = 20, C_d = 10, A = 100, h = 1, r = 0.5, α = 0.05, β = 10, δ = 0.04 (with appropriate units).

We obtain the optimal valuesof t_1 =0.393879006 units, T =0.6366588668 units, Q = 16.5988654 units and Optimal total cost $TC(t_1, T)$ =685.245444 units.

VI. SENSITIVITY ANALYSIS

Sensitivity analysis depicts the extent to which the optimal solution of the model is affected by the changes or errors in its input parameter values. In this section, we study the sensitivity of total cost per time unit (TC) with respect to the changes in the values of the parameters A, a, b, c, α, β, δ, h, r, Cd, Cs, ℓ , pc.

The sensitivity analysis is performed by considering different values in each one of the above parameters keeping all other parameters same and the results are presented in Table – 1.

Table-1
Partial Sensitivity Analysis

Parameter	Values	t_1	T	$TC(t_1, T)$
A	85	0.3686299991	0.5943567319	660.8786638
	90	0.3773483185	0.6089379509	669.1890679
	95	0.3857561316	0.6230251143	677.3059603
	100	0.3938790060	0.6366588668	685.2454440
	120	0.4239285319	0.6873091255	715.4257080
a	10	0.3961355768	0.6404506353	483.1116272
	15	0.3950029853	0.6385472826	584.1742421
	20	0.3938790060	0.6366588668	685.2454440
	30	0.3916564981	0.6329261509	887.3615832
	50	0.3873106594	0.6256324556	1291.559943
b	12	0.4151456207	0.6724690952	665.3819217
	15	0.3938790060	0.6366588668	685.2454440
	18	0.3752175335	0.6053718101	704.0713748
	22	0.3536446799	0.5693551345	727.7982004
	26	0.3351247468	0.5385607156	750.1566725
c	10	0.3938790060	0.6366588668	685.2454440
	17	0.3618325250	0.5830063033	702.6870647
	26	0.3335569871	0.5359591054	721.5459880
	37	0.3091808911	0.4956115169	741.1588099
	40	0.3037579098	0.4866565337	746.0176466
α	0.04	0.3938886295	0.6366595597	685.2428285
	0.05	0.3938790060	0.6366588668	685.2454440
	1	0.3929855073	0.6365946186	685.2498171
	1.5	0.3925309263	0.6365619990	685.2450106
	2.5	0.3916516883	0.6364990364	685.2491020
β	1	0.2576616506	0.6312067650	688.1075333
	3	0.3675461564	0.6351191819	685.5572822
	5	0.3891649564	0.6363480470	685.2728609
	10	0.3938790060	0.6366588668	685.2454440
	15	0.3939267122	0.6366623012	685.2347269
δ	0.025	0.4001464423	0.6363790988	685.3784371
	0.028	0.3989199362	0.6364341425	685.3126224
	0.030	0.3980937980	0.6364710243	685.3024371
	0.035	0.396005271	0.6365642491	685.2804146
	0.040	0.3938790060	0.6366588668	685.2454440
h	1	0.3938790060	0.6366588668	685.2454440
	1.5	0.3354820263	0.6342251066	686.5074420
	2.2	0.2773369547	0.6319490165	687.7213047
	2.5	0.2580644552	0.6312261104	688.1097287
	2.9	0.2361575874	0.6304219108	688.5464814
r	0.5	0.3938790060	0.6366588668	685.2454440
	1	0.3812054726	0.6359771819	685.4374974
	1.5	0.3354820263	0.6342251066	686.5074420
	2	0.3599238849	0.6348729862	685.7660870
	2.5	0.3508314281	0.6344164979	685.9088612
Cd	2	0.3938916734	0.6366597788	685.2429959
	8	0.3938821721	0.6366590948	685.2392093
	5	0.3938869222	0.6366594368	685.2353438
	10	0.3938790060	0.6366588668	685.2454440
	15	0.3938710930	0.6366582971	685.2425719

Parameter	Values	t_1	T	$TC(t_1, T)$
Cs	0.5	0.1428607304	0.6459969140	681.3638350
	1	0.2724485477	0.6415808518	683.2625942
	1.5	0.3460556321	0.6386966403	684.4401676
	2	0.3938790060	0.6366588668	685.2454440
	3	0.4525479190	0.6339607275	686.2762791
ℓ	5	0.3571709910	0.6382350605	684.6187310
	8	0.3693737170	0.6377200514	684.8289948
	15	0.3938790060	0.6366588668	685.2454440
	20	0.4086525836	0.6360010829	685.4944409
	22	0.414038921	0.6357577671	685.5860755
pc	10	0.5434399929	0.8282762959	414.5787058
	18	0.4029793908	0.6362553232	685.4005882
	20	0.3938790060	0.6366588668	685.2454440
	22	0.3732941459	0.6129039230	737.1022933
	25	0.3453632855	0.5820995765	883.0763911

It is observed from Table – 1 that total cost per time unit (TC) is highly sensitive to changes in the value of 'a', moderately sensitive to changes in the values of A, b, c, pc and less sensitive to changes in the values of α, β, δ, h, r, Cd, Cs, ℓ.

VII. CONCLUSION

In this paper, we developed a deterministic inventory model with quadratic demand with respect to time and follows constant deterioration with time dependent holding cost. In this model, shortages are allowed and partially backlogged.Different costs have been illustrated through the numerical example and sensitivity analysis. The obtained results indicate the validity and stability of the model. The model is solved analytically by minimizing the total cost per time unit inventory cost.

REFERENCES

1. Abad, P. L., (1996), "Optimal pricing and lot-sizing under conditions of perishability and partial backordering. Management Science, 42, 1093 – 1104.
2. Chang H.J., Dye C.Y., (1999), "An EOQ Model for deteriorating items with time - varying demand and partial backlogging", *Journal of Operational Research Society*, 50, pp. 1176 – 1182.
3. Covert R.P., and Philip G.C. (1973), "An EOQ model with Weibull distribution deterioration", *American Institute of Industrial Engineers Transaction*, Vol. 5, pp. 323 – 326.
4. Dave U, and Patel L.K. (1981), "(T, Si) policy inventory model for deteriorating items with time proportion demand", *Journal of Operational Research Society*, 32, pp.137 – 142.
5. Dye C. Y., Ouyang LY, Hsieh TP (2007) "Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate." *European Journal of Operational Research*, 178(3), pp. 789-807.
6. Ghare, P. M., and Schrader, G.F. (1963), "A model for an exponentially decaying inventory", *Journal of Industrial Engineering*, 14, pp. 238 – 243.
7. Goyal, S.K., and B.C. Giri (2001), "Recent trends in modeling of deteriorating inventory", *European Journal of Operational Research*, 134, pp.1 – 16.
8. Harris F.W. (1915), "Operations and cost", Shaw Company, Chicago: A. W.
9. Hollier, R.H., and Mak, K.L. (1983), "Inventory replenishment policies for deteriorating items in a declining market", I*nternational Journal of Production* Research, 21, pp. 813 – 826.
10. J. Jagadeeswari, P. K. Chenniappan (2014), "An order level inventory model for deteriorating items with time – quadratic demand and partial backlogging", Journal of Business and Management Sciences, Vol. 2, No. 3A, 17 – 20.
11. Kirtan Parmar & U. B. Gothi (2014), "Order level inventory model for deteriorating items under quadratic demand with time dependent IHC", Sankhaya Vignan, NSV 10, No. 2, pp. 1 – 12.
12. Mishra, V.K,.& Singh, L.S. (2011),"Deteriorating inventory model for time dependent demand and holding cost with partial backlogging", International Journal of Management Science and Engineering Management, 6(4), pp. 267 –271.
13. Nahmias, S., (1978), "Perishable inventory theory: A review", Operations Research, 30,pp. 680-708.
14. Nita H. Shah and KunalT.Shukla(2009), "Deteriorating Inventory model for waiting time partial Backlogging", Applied Mathematical Sciences, Vol 3, No.9, pp. 421-428.
15. Ouyang, L.-Y., Wu, K.-S., & Cheng, M.-C. (2005),"An inventory model for deteriorating items with exponential declining demand and partial backlogging", Yugoslav Journal of Operations Research, 15(2), pp. 277-288.
16. Park, K. S., (1982), "Inventory models with partial backorders", International Journal of Systems Science, 13, pp. 1313 – 1317.
17. Rafaat, F., (1991), "Survey of literature of continuously deteriorating inventory model", Jounal of Operation Research Society, 42, pp. 27 – 37.
18. Sarala Pareek and Garima Sharma (2014),"An inventory model with Weibull distribution deteriorating item with exponential declining demand and partial backlogging", ZENITH International Journal of Multidisciplinary Research, Vol.4 (7).
19. Shah Y. K. and Jaiswal M.C. (1977),"An order-level inventory model for a system with constant rate of deterioration",Opsearch 14, No. 3, pp. 174 – 18.
20. S.R. Singh & T.J. Singh(2007), "An EOQ inventory model with Weibull distribution deterioration, ramp type demand and partial backlogging", Indian Journal of Mathematics and Mathematical Science, 3, 2, 127-137.
21. Singh, T.J., Singh, S.R. and Dutt, R. (2009), "An EOQ model for perishable items with power demand and partial backlogging", International Journal of Production Economics, 15 (1): 65-72.
22. Skouri, K., I. Konstantaras, S. Papachristos, and I. Ganas, (2009), "Inventory models with ramp type demand rate, partial backlogging and Weibull deterioration rate", *European Journal of Operational Research*, 192, pp. 79 – 92.
23. Vinodkumar Mishra, (2013), "An inventory model of instantaneous deteriorating items with controllable deterioration rate for time dependent demand and holding cost", Journal of Industrial Engineering and Management, 6(2), 495 – 506.
24. Whitin, T.M. (1957), "The theory of Inventory Management", 2nd edition, Princeton University Press, Princeton, New Jersey, pp. 62 – 72.
25. Wilson R.H. (1934), "A scientific routine for stock control", Hary Bus Rev 13, pp. 116 – 128.