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Research Paper

A Comprehensive Study on Advanced Sequencing Models and Linked Problems

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Abstract: The present study pertains to distinct categories of flow-shop sequencing problems, which hold significant importance within the domain of Operations Research. The genesis of Operation Research can be traced back to the Second World War, wherein researchers were tasked with analysing strategic and tactical issues pertaining to air and land defence. This gave rise to a novel discipline that proved instrumental in securing victory. The advancement of industrial organisation has led to a rise in the intricacy of managerial functions across diverse domains, thereby posing challenges in the efficient utilisation of scarce resources. The field of Operations Research endeavours to furnish methodologies for a dispassionate approach to predicaments related to decision-making. The present study emphasises the significance of sequencing as a means of enhancing the utilisation of existing facilities and delves into the distinction between sequencing and scheduling.

Keywords: Operation Research, flow-shop sequencing problems, decision-making, optimization, industrial organization, sequencing, scheduling.

1. Introduction

This research work pertains to specific categories of flow-shop sequencing problems, which constitute a significant domain within the field of Operations Research. The nomenclature "Operation Research" was initially introduced in 1940 by Mc Closky and Trefthel in the town of Bowdsey, located in the United Kingdom.

Operation Research originated primarily during the Second World War. During that period, the military administration in England enlisted the expertise of a group of scientists to conduct an analysis of the strategic and tactical challenges associated with air and land defence of the nation. Given their limited military resources, it was imperative to determine the optimal allocation of these resources, such as through efficient ocean transportation and effective bombing strategies.

Therefore, this novel field of study emerged within a military framework. During the Second World War, the military commands of the United Kingdom and the United States of America enlisted the expertise of numerous interdisciplinary teams of scientists to conduct scientific research on strategic and tactical matters. These teams were comprised of individuals from various disciplines and were organised to aid in the resolution of strategic and tactical military operations. The objective of their mission was to develop precise recommendations and strategies to assist Military Commands in determining the most efficient allocation of limited military resources and efforts, and to ensure the successful execution of these decisions. The Operational Research (OR) teams were not actively involved in combat operations or participating in the war effort. However, it should be noted that these individuals served primarily as advisors and their contribution to the war effort was largely limited to the application of scientific and systematic methodologies inherent in the field of Operations Research. Nevertheless, this intellectual support proved to be a valuable asset in facilitating the strategic initiatives of military commands and ultimately contributed to the successful outcome of the war. Therefore, it can be argued that Operations Research (OR) is linked to the concept of achieving victory in a conflict without engaging in direct combat.

The growth of industrial organisation has led to a rise in the complexity of managerial functions across various sectors such as defence, business, industry, and civil government. The swift expansion of industrial operations encompasses a range of determinations pertaining to procurement, strategizing, and overseeing manufacturing, distribution, employment, and related activities. The escalation of activities has resulted in the challenge of optimising the utilisation of finite resources such as labour, equipment, and materials for maximum benefit. The field of Operations Research (O.R.) endeavours to offer methodologies for a rational and impartial approach to addressing decision-making issues. Operations research (O.R.) is a quantitative approach to decision-making that aims to identify optimal solutions to problems that are in the best interest of an organisation. Operation Research problems can be classified into various categories, including sequencing, routeing, allocation, replacement, inventory, queuing, competitiveness, and search.

The issue of allocation can manifest as either a linear programming problem or an assignment problem.

The issues pertaining to fixed facilities and controlled sequencing of customer service have led to the development of a sophisticated sequencing theory. Scheduling and sequencing dilemmas are frequently encountered in various aspects of our daily routines. Examples include arranging the order of jobs for processing in a manufacturing facility, coordinating the landing clearances of aircraft, and determining the sequence of programmes to be executed at a computer centre. Such issues arise when there is a possibility of selecting an alternative sequence for performing a set of tasks. The process of arranging the order or sequence in which waiting customers (or jobs) are served is referred to as sequencing. Similar to other problems in the field of Operational Research, the primary aim is to maximise the utilisation of existing resources in order to efficiently handle the items or tasks at hand.

Choosing a criterion or objective function is a crucial step in the process, as it involves carefully considering the situation and its surrounding environment. Assuming that the duration necessary for organising the processing of a particular item on a designated machine is provided. The issue of optimisation pertains to the arrangement of terms at each facility in order to attain the objective of minimising the total elapsed time, total waiting time, or the cost linked with Operations.

The production of items in various industries involves the participation of man, machine, material, and money. The objective of an industry manager is to optimise the utilisation of resources such as labour, machinery, raw materials, and financial capital in order to minimise production costs and remain competitive in the market.

Deterministic sequencing and scheduling has been a subject of significant interest in academic literature over the past 35 years. The discipline of scheduling can be categorised into distinct domains, including but not limited to single machine scheduling and parallel machine scheduling. This text pertains to the scheduling of various types of shops, including flow-shop, job-shop, and open-shop scheduling.

In recent decades, there has been an increasing focus on stochastic sequencing in academic literature. Scheduling techniques are highly beneficial in industry and business activities, as they provide a scientific and systematic approach to utilising idle machine or operator time in order to reduce production costs. The utilisation of scheduling methods can be extended to other industries or business units in a cost-effective manner.

2. Difference Between Scheduling and Sequencing:

The concept of sequencing pertains to the establishment of a specific order in which jobs are to be executed across multiple machines. On the other hand, scheduling involves the creation of a timetable that encompasses the start time, duration, and finish time of jobs on said machines. The process of scheduling encompasses both the allocation of time slots and the arrangement of tasks in a specific order. Although there are some differences between the terms sequencing and scheduling, they are often used interchangeably in the academic literature.

It is organised into several sections.

1. Classification of scheduling models
2. Performance measure
3. Brief survey of scheduling problems

2.1 Classification of Scheduling Models

Various scheduling problems exist, including:

(a) Flow-Shop Scheduling :

The problem at hand involves a scenario where there are n jobs and m machines. Each machine necessitates execution for any given job. The processing order of all jobs across different machines is uniform, and the progression of jobs through any given subsequence of machines is fixed, such that a job cannot overtake another while awaiting processing on a machine. The aforementioned constraint imposed on a Flow-shop is commonly referred to as a permutation Flow-shop. The permutation flow-shop scheduling problem entails a uniform machine order for all jobs, with the scheduling search being fully defined by a singular permutation of integers 1 through n. This permutation dictates the sequence in which jobs are processed on each machine.

(b) Job-Shop Scheduling Problems:

The problem at hand involves a scenario where there are n jobs and m machines, with each job having a specified order of machines.

(c) Open-Shop Scheduling Problem:

In problems of this nature, it is necessary for each of the m machines to perform a task in order to complete a given job. The sequence in which a task traverses the machinery is inconsequential.

(d) Project Scheduling:

This refers to a type of project that involves comprehensive utilisation of all available resources. This research paper focuses on the Flow-shop scheduling problems.

Basic Assumption in Flow-Shop

- It is a fact that no machine has the capability to execute more than a single operation concurrently.
- Once a machine operation has commenced, it is imperative that it be carried out to its full completion.
- Every operation has a finite duration and it is imperative that it is concluded prior to the commencement of any subsequent operation. The operation time provided encompasses the duration required for both preparation and execution.
- Each type of machine is represented by a singular unit.

(a) Assumption Regarding Jobs

- At time zero, all available job opportunities are ready for processing.
- All occupations follow a uniform sequence of operations.
- Employment opportunities are distinct and not interdependent.
- Each machine processes each job only once.
- Every employment opportunity comprises a predetermined quantity of tasks, and each task is executed by a singular machine.
- The independence of job processing times from the order of job execution is observed.
- It is imperative that every task is carried out until it reaches its conclusion.

(b) Assumption Regarding Operating:

- Every task is executed with utmost promptness.
- Each employment opportunity is deemed as an undividable entity, notwithstanding the possibility of comprising multiple distinct components.
- Adequate waiting space is allocated to each machine to enable jobs to await processing prior to initiation.
- All machines adhere to a uniform job processing sequence, whereby no allowance is made for job passing or overtaking.
- The duration of transportation between machines for a given job is considered to be insignificant.
- The times required for equipment or resources to be prepared or arranged are not affected by the order in which they are performed.
- The operation times provided encompass the duration required for both the setup and execution of the task.

2.2 Various Performance measure

(i) Release time (r_i):

This is the time at which job is released to the shop by some external job- generation process.

It is the earliest time at which processing of first operation of the job could be begin. It is also known as ready time or arrival time of job i.

(ii) Due date (d_i):

It is the promised delivery date of the job i. It is the time when the processing of the last operation should be completed i.e., the time when the job i should be completed.

(iii) Completion time (c_i):

It is the time at which processing time of the last operation of the job i is completed.

(iv) Flow-time (F_i) :

It is the total time that the job i spends in the shop. $F_i = C_i - r_i$

The flow time F_i is also called the manufacturing interval and the shop time.

Note: when r_i=0 then F_i=C_i i.e., completion time of job i is equal to flow time of job i.

(v) Lateness (L_i):

It is simply difference between completion time of job i and its due date.

(vi) Tardiness (T_i):

The tardiness of job i is equal to Max {0,L_i}

(vii) Earliness (E_i):

The earliness of job i = Max {0,-L_i}

Note: The concepts of lateness, tardiness, and earliness involve the comparison of the actual time of completion with the intended time of completion.

The concept of lateness involves the calculation of the algebraic difference between each job, irrespective of the sign of the difference. The concept of tardiness only takes into account the time difference between the expected arrival time and the actual arrival time, without considering any negative impact that the delay may have caused. That is to say, tasks that are finished past their designated deadline are regarded as tardy, while those completed ahead of schedule are only evaluated based on the negative deviation from the expected completion date. In the context of project management, it can be observed that a job's timeliness can be evaluated by a metric called Li. Specifically, when a job is completed prior to its due date, Li assumes a negative value. Conversely, when a job is completed after its due date, Li assumes a positive and non-zero value, indicating that the job is tardy.

(viii) Total Elapsed Time (C_{max}):

It defined as total completion time in which the set of all the jobs finish processing on the machines. It is also known as make span.

C_{max} = Max {C_i}; i= 1,2,3,4 ,n.

(ix) Idle Time (I_k):

It is idle time on machine M_k. $L_k = C_{max} – P_{i,k}$, where $P_{i,k}$ is the processing time of job I on machine Mk.

(x) Mean Completion Time (C):

It is the average completion time of any job C = (C_i)/n.

(xi) Mean Flow Time (F):

It is the average time a job spends in the shop. C = (F_i)/n

(xii) Penalty Cost :

The term refers to the aggregate amount of fines incurred due to the delayed completion of tasks beyond their scheduled deadlines.

(xiii) Total Production Cost:

The term refers to the aggregate cost incurred in the manufacturing process of a specific set of products utilising machines.

Two performance measures are considered equivalent if a schedule that is optimal in terms of one performance measure is also optimal in terms of another performance measure, and vice versa. The equivalence between performance measures C and F has been established, whereas the non-equivalence has been observed between measures C_{max} and F_{max}.

Parameters in Scheduling Problem

The scheduling problem involves four parameters, namely n/m and A/B.

n: the number of jobs

m: the number of machine

A: describes the flow patterns or discipline within the machine Shop.

F: for the flow shop case i.e., the machine order for all the jobs is the same.

P: for the permutation flow shop case.

G: for the general job-shop case when there are no restrictions on the form of the technological constraints.

B: describes the performance measure by which the schedule is to be evaluated. As an example, $n/2/f/C_{max}$ is the n-jobs, 2 machines flow-shop problem where the objective is to minimize total elapsed time.

Terminologies

(a) No-idle constraint:

In the absence of any idle constraints, machines operate in a continuous manner without any intervals of idleness. Specifically, once a machine initiates the processing of its first job, it will continue to operate continuously without any interruptions until the final job has been completed on it. This implies that a given machine operates for a duration equivalent to the total processing times of all jobs assigned to it.

(b) No wait Constraint:

The absence of wait constraints necessitates uninterrupted job processing across consecutive machines, whereby a job must traverse a series of machines without any inter-operation delay. The aforementioned statement denotes that the duration between the finalisation of the last operation of a task and the initiation of its initial operation is equivalent to the summation of the task's operation durations across all machines.

(c) Non-Availability Constraint:

The majority of literature pertaining to scheduling operates under the assumption that machines are continuously available. Nonetheless, the accessibility of such resources may not necessarily hold true in practical industrial environments. During the scheduling period, machines may be unavailable due to stochastic breakdown or deterministic preventive maintenance. It is possible that the machines may not be accessible at the onset of the planning period, as they may still be engaged in processing incomplete jobs that were previously scheduled.

(d) Pre-emption:

The scenario at hand involves a multifaceted circumstance wherein the processing procedure is disrupted and a task is extracted from a device prior to the culmination of an operation. According to scholarly discourse, pre-emption takes place when the execution of a task that has been initiated is terminated prior to its conclusion. In the context of pre-emption, it is inadequate to represent a schedule of n jobs using a sequence of n integers, as the schedule may contain multiple occurrences of individual jobs and it is necessary to determine the duration of each appearance. Various forms of pre-emption can be observed, contingent upon the manner in which the suspended task is handled upon its subsequent return to the system for additional processing. At the furthest end of the spectrum, the processing can seamlessly resume from the point of interruption without incurring additional effort or time. This practise is commonly referred to as pre-emptive resumption discipline. On the opposite end of the spectrum lies the pre-empt-repeat approach, wherein any processing that has been executed is rendered null and void upon interruption, necessitating a repetition of the processing once the task is returned to the machine. The minimum processing time for the job was stated to be achievable only if the schedule allows for uninterrupted processing of the jobs. A highly rational and significant practise that lies between these two extremes entails preempting resume operation during the processing time and preempting it again during the setup time.

(e) Inserted Idle Time:

This refers to a scheduling strategy where a machine remains inactive despite the presence of pending work that needs to be completed. In flow-shop sequencing problems where there is a singular machine available for each task, the positioning of said machine is entirely determined by the sequencing choices made by the job. When multiple machines of the same type are available for processing a set of jobs, the scheduling process involves both job sequencing and machine allocation for job processing, as outlined in the journal. The field of study pertaining to the arrangement of tasks on multiple machines is commonly referred to as parallel machine sequencing in academic literature.

These classes are P, NP, and NP-complete can solve them efficiently or not (i) identical (ii) uniform (iii) unrelated.

Assuming machine homogeneity, the processing duration of task I remains constant across all machines.

In cases where the machines exhibit uniformity, the duration of processing for job I on any given parallel machine M_k can be expressed as P_i/S_k. Here, P_i represents a fixed value for job I, while S_k denotes a speed factor that is linked to machine M_i. In cases where machines lack a relationship, the processing time for job I may vary across parallel machines in a non-deterministic manner.

(f) Shortest Processing Time (SPT):

The act of arranging tasks in ascending order of processing duration is commonly referred to as shortest processing time (SPT) sequencing, or alternatively, shortest operation time. The demonstration of optimality is facilitated by a valuable technique known as the method of adjacent pairwise interchange.

Mean Flow-Time (F) is minimized by SPT sequencing ($p_1 < p_2 < \ldots\ldots\ldots\ldots < p_n$), where p_i is the processing time of job i.

Weighted Mean Flow-Time (Fn) is minimized by WSPT sequencing ($p_1/w_1 < p_2/w_2 < \ldots\ldots < p_n/w_n$); where w_i is waiting factor to each job.

2.3 Brief Survey of Scheduling Problems

During the early stages of industrialisation, engineers and consultants in the field of management were actively engaged in addressing various issues related to contemporary manufacturing and production control. The topic of work flow within a factory was explored by Knoeppel in 1915, while Gantt in 1919 provided an explanation of the utilisation of visual charts. In 1928, Coestt presented a mechanical scheduling technique that exhibited similarities to the contemporary kanban system. Several other approaches were also developed to address this issue. Over the course of the last 40 years, scheduling has been the subject of research from a mathematical perspective, as evidenced by the involvement of the Operation-Research, Operations Management, and Artificial Intelligence communities. Within these communities, scheduling is regarded as a process of sequencing. Specifically, the scheduling of production systems involves the allocation of a group of machines to execute a series of work-orders within a designated timeframe. In essence, scheduling pertains to the arrangement of operations across multiple machines.

In 1954, Johnson successfully employed a heuristic algorithm to obtain an optimal solution for the n jobs 2 and 3 machines flow shop problem. This was achieved by considering the processing time on each machine, while adhering to various constraints. Mitten (1959) addressed cases that were comparatively less restrictive, thereby rendering them applicable to a broader range of problems. Wagner (1959) and Conway et al. (1960) developed integer programming models to address scheduling problems. Ignall and Scharge (1965) employed the branch and bound method. The economic aspect of scheduling theory was examined by Gupta in 1969. Various heuristic methods are employed in the scheduling of job shop and flow shop operations. Furthermore, the investigation of diverse scheduling procedures and their influence on job performance has been conducted under relatively uniform circumstances using computer programmes.

Maggu and Das (1977) proposed the equivalent job-block theorem for 2×n sequencing problems. The processing of jobs was examined by Maggu et al. [1981] with a focus on the breakdown effect.Singh T.P. (1985, 1986, 1998) expanded upon the work, incorporating various parameters such as transportation times, arbitrary times, set up times, and break down intervals. Singh T.P. (1992-94) conducted a study utilising computer technology to explore the sophisticated applications of queuing and scheduling models in various industries. The utilisation of Variable Range was employed by Ching Jong Liao and colleagues in their study conducted in 1992, which aimed to address the issue of multiple criteria scheduling problems.

In 1990, McCahon and Lee utilised the fuzzy set theory to analyse the performance characteristics of a flow system. Petrovic and Song (2003) employed a cut-based approach within a fuzzy environment to address the flow shop problem.

The study conducted by J. LIV and B.L. MacCarthy in 1999 focused on examining heuristic procedures and solution strategies that are applicable to flexible manufacturing systems (FMS). The works of Suresh V and Deepak Chandhauri in 1996, as well as Bagga P.C. et al. in 2000, are noteworthy contributions to the field. Anup and Maggu's study was conducted in 2002. Singh and Gupta are two individuals whose full names are not provided. Deepak and colleagues (2004-2005) conducted a study on the optimal scheduling of two and three stage production processes. The study considered the probabilities associated with processing time and set up time, as well as job-block criteria. In 2006, Narain L. examined specific instances of flow-shop models that involve biobjective scheduling problems, utilising the Branch & Bound technique. Further Singh, T.P. and Gupta. Deepak (2006) endeavoured to reduce the rental expenses of machinery by utilising a straightforward heuristic approach that incorporated various parameters. The heuristic search algorithm for flow-shop scheduling was investigated by Joshua Poh-Onn Fan and Graham K. Winley in 2007. Vijay Singh [2011] devoted significant effort to examining three flow-shop scheduling problems involving machines, with a focus on the total rental cost. Further The research conducted by Deepak Gupta [2011] focused on the Minimisation of Rental Cost in a Two Stage Flow Shop Scheduling Problem. The study involved the separation of Setup Time from Processing Time, with each being associated with Probabilities, including Job Block Criteria.

3. Conclusion

The field of Operations Research is a crucial area of enquiry that facilitates the optimisation of decision-making processes across diverse domains. The present study examines distinct categories of flow-shop sequencing problems and underscores the significance of sequencing as a means of enhancing the utilisation of existing resources. Distinguishing between sequencing and scheduling is a pivotal factor in devising effective resolutions for such quandaries. The application of Operation Research techniques offers a systematic and impartial methodology for resolving decision-making challenges, with the ultimate goal of proposing optimal solutions that align with the organization's best interests. The findings of this study can provide valuable insights for managers to make informed decisions and optimise the utilisation of scarce resources, thereby reducing expenses and enhancing efficiency.

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